

English Language Learners
&
The
Language
of
Mathematics

Bilingual Education Division
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INTRODUCTION TO THE LANGUAGE OF MATHEMATICS

Johann Wolfgang von Goethe, the German poet and dramatist, controversially said:
“Mathematicians are like Frenchmen. Whatever you say to them, they translate into their own language, and forthwith it is something entirely different.”

While this is somewhat of an exaggeration, there are elements of truth in this statement. Mathematicians have long acknowledged that they employ a variety of English that differs substantially from its common usage by the general public. This language of mathematics:

- a. has its own specialized vocabulary of numbers, symbols and words;
- b. employs agreed upon definitions for these numbers, symbols and words that are never vague nor implied but which are specific to the discipline;
- c. follows regularized rules of how these symbols or words may be used in verbal and mathematical sentences
- d. conveys meanings that can be communicated with these numbers, symbols and words
- e. helps communicate mathematic concepts that are clear and precise;
- f. develops and presents an argument that is accurate, definite, stable, and never ambiguous;
- g. is always exact, never ‘close’ for the argument it is making to be useful
- h. allows shorthand communication by a community of people who regularly use these numbers, symbols and words.

Curriculum specialists and teachers recognize the need for students to understand and use communication in mathematics. They realize that students need to read and make sense of mathematical situations. Students need to interpret these situations both as mathematical statements and also as verbal expressions of mathematical conditions. This specialized language not only represents and describes mathematical circumstances, its use aids in thinking about and comprehending increasingly more and more abstract problems, algorithms, and relations. Students need to learn how to “do” math. And, they must also learn how to articulate they did and what they are learning.

The language load in mathematics instruction is extensive. As new operations and concepts are introduced, the language of mathematics increases dramatically. Even at kindergarten and first grade, students are inundated with new vocabulary, mathematic notation and verbal expressions that accompany each lesson. An example of this is found in Appendix I. Students needs to understand the teacher’s explanations, make sense of examples provided, follow suggested procedures and be able to ask questions and talk how they solved the posed problem. These require facility with the language of mathematics.

The National Council of Teachers of Mathematics (NCTM) recognized this language and its role in our curriculum when they recommended a K-12 Communication Standard to enable students to ‘use the language of mathematics to express mathematical ideas precisely’.¹ NCTM also recognizes that students will think, perform and communicate mathematically once their teachers model these behaviors for them. The language of mathematics is central to this effort.

¹ *Principles and Standards for School Mathematics*. 2000. National Council of Teachers of Mathematics.

This manual is designed to introduce teachers to the language of mathematics and its role in facilitating the organization, analysis and evaluation of thinking in mathematics as well as its use in communicating mathematical ideas to others. This manual will begin with a discussion of the elements of this language and a listing of some points of confusion between it and informal English. Next, it will enumerate information on pronunciation (proper reading of mathematical expressions) and translation (between verbal and mathematic notation) of mathematical statements. Finally, this document will share some insights for teaching this language both to fluent native English speaking students as well as to those who are acquiring English as their second language.

ELEMENTS OF THE LANGUAGE OF MATHEMATICS

The Vocabulary Of Mathematics

Mathematics lessons and texts are designed to provide students with hints and rules to help them master concepts and procedures and to use algorithms. They provide models for students to practice in demonstrating procedures, responding to exercises, and solving problems. Just as importantly, students will need to become familiar with all information, definitions, theorems and notation pertinent to the concept being studied. Language is central to the study of mathematics as it helps carry meaning of its ideas and practices and it assists in their communication with others.

Mathematics is foremost about numbers and how we represent them. Any consideration of number quantities should include those words and figures which represent positive and negative integers, decimals and fractions from zero, 1, 2, 3, to infinity used in counting, calculation and measurement. In addition, mathematics uses words and figures to represent relationships in the systematic study of structure, space and change. There are, to be sure, thousands of terms that are used in mathematics. Many of these are listed in appendix II.

Meanings specific to mathematics

Similar to other professions, mathematics also has its own brand of technical terminology. In some cases, a term used within mathematics has a specific meaning that differs from the way it is used in informal nonmathematical communication. Examples include right, left, group, point, ring, perimeter, field, category and feet.

Meaning only in mathematics

In other cases, mathematics uses terms that don't exist outside of mathematics including denominator, numerator, equivalent fraction, tensor, square root, pi, fractal, functor, exponent, hypotenuse, sine and cosine.

Multiple terms

Although the language of mathematics is thought to be exact and accurate, at times multiple terms are used to signal a single transaction. For example in addition, the operation is indicated by the following terms: and, plus, added to, increased by, combined with, raised by, more, gains, total of and sum of. In subtraction, equivalent terms include: less than, minus, decreased by, separated from, take away, loss, fewer, difference between and subtract. Likewise, times, multiplied by, multiples of, product of, double, triple, percent of and fraction of are heard in multiplication and half (or third) of, separate equally, per, quotient of, divided by, and divided into are common in division.

Pronouns

One difference between the language of mathematics and informal English involves the presence of 'pronouns'. For example, 'x' in a mathematical sentence can represent an unknown quantity as in $x+3=5$. It can also be a placeholder for an unspecified value in mathematical generalization which asserts that something is always true. For example in the statement "for all x , $3(x+2) = 3x + 6$ ". This may be abbreviated as $3(x + 2) = 3x+6$. One important aspect of these pronouns is that in the language of mathematics the placeholder or unspecified value is case sensitive. That is, capital and lower case letters used to represent them are not interchangeable so that $a \neq A$ and $X \neq x$.

Common misused terms

Even common informal English terms can cause difficulty if misunderstood or misused by mathematicians and students of mathematics. In the language of mathematics, "the" indicates one such object while "a" or "an" suggests one of many. "Of" is used when information about one item is about to be stated (ie. 5% of ___ = ?) and "of" also is used to indicate proportion such as in 3/4ths of a pizza. "Only" and "or" have a more precise meaning in the language of mathematics than is the case with regular English. Likewise, use of the word "by" can communicate four different operations ["reduce by 5" (-); "3 multiplied by 7" (x); "7 increased by 2" (+); and "21 divided by 7" (\div)].

"is"

We also find that the very common mathematical verb "is" can signal different meanings ranging from:

- "equivalent to" (4 is the square root of 16),
- "specification of a power as an adjectival phrase that may or may not exist" (e.g. 4 is less than 20), or
- "an example of" (3 is a prime number).

When these three examples are written using mathematical notation, their meaning becomes clearer:

- $4 = \sqrt{16}$
- $4 < 20$
- not normally written symbolically but can be represented as an element of a set of numbers $\in \{2, 3, 5, 11, 13, 17, 19\}$

Equals sign

Related to "is" are the various interpretations of the "=" sign. In elementary grades mathematics instruction, the "=" may communicate that the answer to a problem written on its left is to be followed by an answer written on the right side of the "=". (As in $2+5=$ ___). Sometimes a pattern provided on the left side of the "=" is assigned an equal value on the right side. Another interpretation is that values on both sides of the "=" are equivalent, symmetrical, or the same. A third interpretation distinguishes between equality and equivalence. Equality is present in $3+8=11$. Equivalent expressions result when equal numbers are produced with different sequences of operations [e.g. $x-(-3)$ and $x+3$]. Equal signs relate two expressions (numbers or variables) but never two sentences (equations). A fourth use of the "=" is for definition. An equals sign can also signal an intention in assertion and proof. = always means equal or "same as".

Stock phrases

Finally, stock phrases such as "if and only if", "necessary and sufficient," "without loss of generality," and "equal to or less than..." have specific meanings that are unique to mathematics.

Mathematical Notations

Mathematical notation has assimilated symbols from many different alphabets and fonts. It also includes symbols that are specific to mathematics. Here are a few of the common mathematic symbols listed in Wikipedia. The list is not all inclusive but represents those symbols which are basic to mathematics:

MATHEMATIC SYMBOLS

SYMBOL	NAME	READ AS	EXPLANATION
=	equals	is equal to; equals	Represent the same value
≠ <> ! =	Inequation or not equal		Two items or numbers do not represent the same value
> >>	Strict inequality	Greater than Much greater than	$x > y$ x is greater than y $x \gg y$ x is much greater than y
< <<	Strict inequality	Less than Much less than	$x < y$ x is less than y $x \ll y$ y is much greater than x
≤	Inequality	Is equal to or less than	$x \leq y$ x is less than or equal to y
≥		Is equal to or greater than	$x \geq y$ x is greater than or equal to y
+	Addition	Plus	Combining of two quantities
-	subtraction	Minus, less	Opposite of addition
x •	Multiplication	Times	One value multiplied by another
÷ /	Division	Divided by Quotient of	$6 \div 3 = 2$ the division of 6 by 3
≈	Isomorphism	Approximately equal	$x \approx y$ both are about equal
√	Complex square root	Square root of...	The divisor of a quantity that when squared gives the same quantity
$f: X \rightarrow Y$	Function notation	The function of (x)	$y = f(x)$, read "y equals f of x," means that a dependent variable y is a function of the independent variable x.
¬	Logical negation	not	The statement $\neg a$ is true if and only if a is false
∞	proportionality	Is proportional to	$x \propto y$ means that $y = kx$ for some constant k
∑	Summation	Sum over ... from 1 to 2 of	$\sum_{k=1}^n a_k$ Means the sum of $a_1 + a_2 + \dots + a_n$
±	Plus-minus	Plus or minus	An equation has two solutions 6 ± 3 means both $6+3$ and $6-3$
...	Absolute value	Absolute value of	$ x $ means the distance along the real line (or across the complex plane) between x and zero.
	Divides	Divides	$a b$ means a divides b
=>	Material implication	Implies; if... then	If $x=2 \Rightarrow$ (it follows then that) $x^2=4 \Rightarrow x=2$ is in general false (since x could be -2)
{, } { }	Set brackets	The set of...	$\{a, b, c\}$ means the set consisting of a, b, and c
()	Precedence grouping	Parentheses	Perform the operations inside the parentheses first

There are hundreds more symbols common to arithmetic, algebra, geometry, trigonometry, and calculus.

Pronunciation in mathematics

The language of mathematics can be written either as mathematical notation (numbers and symbols) or as verbal expressions. Both can be translated from one to the other. This translation between each has to be exact, always clear and never in doubt for it to communicate accurately.

Pronunciation involves how we read mathematical expressions. Poor pronunciation can result in ambiguous or erroneous results.

FOR EXAMPLE, the student pronunciation “a plus b over c” might be used for either

“(a + b)/c”

OR

“a + (b/c)”

To avoid this problem, the student can say:

The quantity “a + b, the quantity divided by c”

“a + the quantity b over c”

“a + b (long pause) over c”

“a + (a long pause) b over c (quickly)

“a + b all over c”

Common mathematical phrases

The following is a listing of verbal phrases and the associated pronunciation of the math expressions for addition, subtraction, multiplication, and division operations. The keywords for operations are underlined.

VERBAL PHRASE	EXPRESSION
ADDITION	
The <u>sum</u> of six and a number	$6 + x$
The sum of six and a number is	$6 + x =$
Eight <u>more than</u> a number	$y + 8$
A number <u>plus</u> five	$n + 5$
A number <u>increased by</u> seven	$x + 7$
VERBAL PHRASE	EXPRESSION
SUBTRACTION	
A number <u>decreased by</u> nine	$n - 9$
The <u>difference between</u> 5 and a number	$5 - y$
4 <u>less than</u> a number	$y - 4$
7 <u>minus</u> a number	$7 - y$

VERBAL PHRASE	MULTIPLICATION	EXPRESSION
The <u>product</u> of 9 and a number		$9n$
10 <u>times</u> a number		$10n$
A number <u>multiplied</u> by 3		$3n$
One fourth <u>of</u> a number		$\frac{1}{4}n$
<u>Twice</u> a number		$2n$

VERBAL PHRASE	DIVISION	EXPRESSION
The quotient of a number and 6		$n/6$
7 <u>divided</u> by a number		$7/n$
5 <u>into</u> 25		$25/5$
One-half <u>of</u> twenty		$\frac{1}{2}(20)$

At a more sophisticated level, we express mathematic expressions as follows²:

Mathematic Notation	Verbal Expression
$5x$	Five x (avoid "five times x")
ab	ab (avoid "a times b");
$x + 2$	x plus two
$x - 2$	x minus two
-5	Negative five [or less accurately] minus five. <i>The temperature is negative 5°.</i>
$5 - (-2)$	Five minus negative two [or] five minus two
$3x + 2$	Three x plus two [avoid "three times x plus two"]
$3(x + 2)$	Three times the quantity x plus two [or] Three times (pause) x plus two
In spoken mathematics, parentheses are hard to say aloud. "The quantity" is a phrase used to alert the listener to parentheses. Also, pauses can be used to try to indicate parentheses.	
$x/2$	x divided by two a number divided by two
$\frac{a + b}{c}$	a plus b, all divided c [or] a plus b, the quantity, divided by c
$3x^2$	three x squared , 3 times x squared, Three times a number multiplied itself
$(3x)^2$	the quantity three x , squared [or] three x, all squared [or] the quantity three x (pause) squared
Mathematic	Verbal Expression

² Adapted with permission from Warren W. Esty. *The Language of Mathematics*. 2006. p. 14.

Notation	
$\sqrt{2}$	the square root of two [some say "root 2"]
$\sqrt{x+2}$	the square root of the quantity x plus two
=	equals [or] is equal to [not "equals to"]
$x + a = b$	x plus a equals b
<	Is less than [avoid "is smaller than"]
$a < b$	a is less than b
\leq	is less than or equal to
>	is greater than [avoid "is bigger than"]
\geq	is greater than or equal to
$-2 < x < 2$	X is between -2 and 2 $X > -2$ and $X < 2$ Negative two is less than x is less than two [or] Negative two is less than x and x is less than two [or less accurately] Minus two is less than x is less than two
$ x $	the absolute value of the quantity x [some say "absolute x"]
$ x + 2 $	the absolute value of the quantity x plus two
$ x + 2$	the absolute value of x (pause) plus two
x^3	x cubed [or] to the third [or] x to the third power
x^p	x to the <i>pth</i> [or] x to the p
ax^2	A times x squared

Mathematic sentences

As a language, mathematics is generally written in conceptually dense statements that can be declared as true or false. An example of a true statement is $(7 \cdot 8 = 56)$. One which is false is $(0 \div 6 = 6)$. Sentences can also be open (without a evaluation) or closed (with an equivalent value specified). Its syntax resembles declarative English **sentence structure** with subjects, verbs and objects along with modifiers that can alter and change their meanings.

A **math fact** can be a number or set of numbers such

5 oranges or 5 sets of oranges, $8x$, or a set of 9 tiles

TRANSLATING MATHEMATICAL/VERBAL EXPRESSIONS

A first step in solving a mathematics problem is to read it carefully. Often, students will need to translate the math symbols into verbal phrases and expressions or vice versa. Just as individuals translate from one language into a second they can also translate words into mathematic numbers and symbols. By focusing on keywords, students can determine the operation and the order of the expression in solving math problems.

Students need to 'translate' between verbal expressions and mathematical notation accurately:

$6x = 7$ is translated and read as "six times an unknown value is equal to seven.

$\frac{1}{2}(4+6) = 5$ is translated as one half of the quantity of four plus six is five.

ADDITION KEYWORDS

Sum of or total of are a few of the keywords that signal an addition operation. The placement of the + is indicated by the "and," "plus," "increased by," "gains," "raised by," and "more' in numeric and word problems.

For example:

The sum of five and six; (addition) $5 + 6$
or, five is increased by six;
or "add six more to the five you already have.

The total of a number and negative two. $x+(-2)$
(addition) unknown+ (-2)

$x^2 + 7 = ?$ x squared plus seven

A word problem: Janie weighed 55 pounds as a second grader and added another 15 pounds in third grade. Write a math expression to show how much she weighed in third grade. Note, the student is asked only to translate the word problem to a numeric expression not solve it.) $55 + 15$

SUBTRACTION KEYWORDS

“Difference between” signals a subtraction operation and the word “and” indicates where the “-” sign is placed. Other subtraction words such as “minus,” “decreased by,” “less,” “fewer,” “take away” along with “and” also indicate where the “-” sign is placed.

For example:

The difference between seven and two is written $7 - 2$

Thirty seven minus five is written $37 - 5$

Twenty five decreased by ten is written $25 - 10$

Seven less three is written $7 - 3$

A store had \$250 invested in candy and sold \$25 worth of candy. $\$250 - \25

Write a math expression showing how much is now invested in candy.

MULTIPLICATION KEYWORDS

Product of and multiply by are phrases that signal multiplication. Other words such as times, percent of, fraction of, double and triple indicate placement of the or multiplication operation symbol.

For example:

The product of three and seven is written 3×7 or $3(7)$ or $3 \bullet 7$ or $3 * 7$

Twenty percent of fifty is written $20\% \times 50$ or $.2(50)$ or $.2 \bullet 50$

The cost of the groceries was \$15.75. $(\$15.75)(.0975) + \$15.75 \times$

The sales tax is 9.75% of the total. or

What was the total cost of the groceries including the sales tax? $(1.0975)(\$15.75)$

DIVISION KEYWORDS

Quotient is one keyword indication division. Other words signaling placement of the \div include divide ___ by (\div) ___; also “and,” “by” and “per.”

For example:

$$\frac{55}{11}$$

Fifty five divided by 11

The quotient of thirty three and eleven

$$\frac{33}{11}$$

TURNABOUT WORDS

The language of mathematics also includes terms which signal that expresses are written in a different order than written or spoken.

Addition

Words that indicate a change in the order of the addition operation from the original English include: “to,” “from,” and “than.”

For example:

Add five to three is written $3 + 5$

Eight added to two is written $2 + 8$

Five more than “x” is written $x +$

Subtraction

Turnabout words for subtraction include: subtract ___ from ___, ___ subtracted from ___, and ___ less than ___.

Examples include:

Subtract seven from fourteen is written $14 - 7$

Twelve subtracted from twenty four is written $24 - 12$

Nine less three is written $9 - 3$

Three less than nine

Multiplication turnabout words

There are no turnabout words in multiplication operations. There is common agreement that the coefficient precedes the variable (as in $8x$ with eight being the coefficient). $8x$ also implies multiplication of 8 times x .

Division turnabout words

“Divide ___ into ___” and “___ divided into ___” are division turnabout words.

Two examples follow:

Divide seven into forty nine is written $49 \div 7$

Eight divided into twenty four is written $8/24$

or

$24 \div 8$

ORDER MATTERS

As a language, mathematics is designed to express thoughts about operations such as addition, subtraction, multiplication, or differentiation and the order in which they should be executed. The order does not necessarily follow the left-to-right order normally followed in reading. When an expression contains more than one operation, the evaluation or solution of a problem can result in different answers depending on the order in which they were solved. Mathematicians have agreed to an order for evaluating expressions to avoid this concern. The rules are:

1. *parentheses* in expressions indicate that the enclosed operation is carried out first, no matter what operations they enclose.
2. operations expressed with *exponents* such as squaring and derivation of square roots are conducted before dealing with negative signs; and, both of these are carried out before *multiplication* or *division* occurs. If you take care of exponents, do it everywhere.
3. multiplication and division operations are performed from left to right followed by addition and subtraction operations.
4. Addition and subtraction left to right.

Potential misreading:

Without parenthesis: $9 - 7 + 3$

$$2 + 3$$

Solution

$$5$$

With parenthesis: $9 - (7 + 3)$

$$9 - 10$$

$$-1$$

ORDER OF OPERATIONS

	Example 1	Example 2	Example 3	Example 4
Initial problem	$15 - 7 + 4$	$5 - (3 + 1)$	$3^2 \times 18 + 2 - 12 + 4$	$17 - 12 + 6$
First operation	$8 + 4$	$5 - 4$	$9 \times 18 + 2 - 12 + 4$	$5 + 6$
Second operation			$9 \times 9 - 3$	
Third operation			$81 - 3$	
Solution	12	1	78	11

MATHEMATICS DISCOURSE

Mathematics discourse refers to the way language is used in mathematics by various communities (mathematicians, teachers, students, etc.). Math discourse includes how sentences or groups of sentences or paragraphs function as textual units, each with a specific meaning and purpose. This discourse can vary by genre such as in explanations, proofs, and presentations but in all cases share the following commonalities:

- Modes of argument such as precision, brevity, and logical coherence are valued
- Different math communities value activities such as abstracting, generalizing, search for certainty, and being precise, explicit, brief, and logical.
- Mathematical claims apply to precisely and explicitly defined set of situations. For example, multiplication results in a larger number unless the multiplying by a number less than one.
- Generalizations are valued as they help us perceive and use repeated patterns of information
- Imaging, visualizing, hypothesizing, and predicting are also valued mathematical practices

Student's participation in mathematical discourse is not merely a matter of learning vocabulary or reading mathematic notation. By engaging in conversations in mathematics, students learn communicate about and negotiate meaning of mathematical situations.

While participating in mathematical discourse, students are often expected to³:

- listen to, respond to, and question the teacher and one another
- reason, make connections, solve problems, and communicate
- describe patterns
- initiate problems and questions
- conjecture and posit solutions
- investigate conjectures by exploring examples and counterexamples
- make generalizations
- explain a concept, justify an answer or elaborate on an explanation or description
- convince themselves and others of the validity of particular representations, solutions, conjectures and answers
- use representations to support claims
- rely on mathematical evidence and argument to determine validity.

Students are frequently asked to show their work and describe their process. This requires students to describe or list the steps they follow beginning with the question in the assignment to the intervening calculations that lead to the answer. Students are expected to use complete English sentences when the meaning of the mathematical sentences is not otherwise clear. Finally, students explain their reasoning and make their computations clear. The following seven step sequence is suggested when students are explaining how they solved a problem:

³ The Professional Standards for Teaching mathematics. National Council of Teachers of mathematics. 1991.
<http://standards.nctm.org>.

STEPS IN SOLVING A PROBLEM

1. State the problem to be solved along with the specification of the proof.
2. Verbally explain the answer to the problem balancing the use of words and symbols and employing transitional phrases to help guide the reader and the listener.
3. Discuss any assumptions made and formulas used.
4. Carefully, precisely and in a well-organized manner explain how the problem was approached and the answer found.
5. Summarize each clearly labeled diagram, picture, table, graph, etc.
6. Define terms, notation and variables. Explain formulas and their derivation.
7. Acknowledge derivations of formulas and explain where each can be found.

Mathematics is a study of pattern and order. It employs numbers, symbols and words to describe form, chance, and change. Mathematics is also a science of abstract objects which relies on logic as its standard of truth. Mathematical statements have their own moderately complex reasoning taxonomy that can be divided into:

- a. **axioms** – or undemonstrated proposition concerning an undefined set of elements, properties, functions and relationships resulting from
 1. deductive reasoning – based strictly on the rules of logic where conclusions are validated using only previously, deductively established results; the definitive valuation of a result
 2. inductive reasoning – process of drawing general conclusions about all future observations from limited observations.
- b. **inferential reasoning** – process of deriving generalizations from statistical data.
- c. **conjectures** – a synonym for ‘statement’ but without any connotation of truth or falsehood
- d. **theorems** – a mathematical result that has been proven
- e. **parameter** – a “constant” fixed for a given application but which varies from application to application.
- f. **proposition** – a matter of fact statement to be demonstrated (less complicated than a theorem)
- g. **lemmas** – a subsidiary proposition assumed to be valid and used to demonstrate a principle proposition; and
- h. **corollaries** – a proposition that follows with little or no proof from one already proven.

Exploring these types of thinking and communication is beyond the scope of this paper but are important, nevertheless, in mathematics instruction.

CONCLUSION

Mathematics can be a frustrating, frightening and often mysterious discipline to many students. Guiding students as they become familiar with and able to use the language of mathematics efficiently can go a long way toward helping make it a remarkable, intriguing, fun, and enlightening subject to study.

MATH and SECOND LANGUAGE LEARNERS

Many English language learners not instructed in dual language immersion or native language classes have been placed in English mathematics classes with the belief that mathematics was language neutral and therefore not dependent on English competence. These beliefs often considered mathematics a discipline concerned primarily with computation and following algorithms in solving numeric problems. The pattern was for students to listen, copy, memorize, and then drill. Today, our understanding of mathematics has changed. Becoming proficient with mathematics requires more than just computation, it involves being able to mathematically explore, represent, explain, investigate, formulate, predict, conjecture, discover, develop, solve, construct, describe, justify, verify, and use patterns and order in our real and virtual world. Each of these verbs in mathematics requires a student to make sense, reason, and think. Students now actively, test ideas, make conjectures, develop reasons and offer explanations which require an understanding and use of the language of mathematics.

John A. Van de Walle in his *Elementary and Middle School Mathematics – Teaching Developmentally* says (p. 14)

Every idea introduced in the mathematics classroom can and should be completely understood by *every* child. *There are no exceptions.* There is absolutely no excuse for children learning any aspect of mathematics without completely understanding it. All children are capable of learning all of the mathematics we want them to learn, and they can learn it in a meaningful manner in a way that makes sense to them.

Mathematics employs a complex, highly precise, vocabulary-rich, sophisticated language that could be confusing to many who already speak English as their first language. and more so of students who are learning English as their second language. They, along with native English speaking students, are deserving of assistance in understanding and using the language of mathematics.

Educators are beginning to understand that with mathematics instruction for student who are learning English, we have the English language learners (ELL) first language (L_1), the new second language, usually English (L_2), and the content specific language of mathematics (L_3). The language of mathematics has its own registry, its own lexicon and syntax, and it communicates complicated operations, measurements, and relationships. Students certainly warrant deliberate instruction, clarification, and guidance with language in order to do calculations, reason, and communicate.

MAKING MATHEMATICS COMPREHENSIBLE

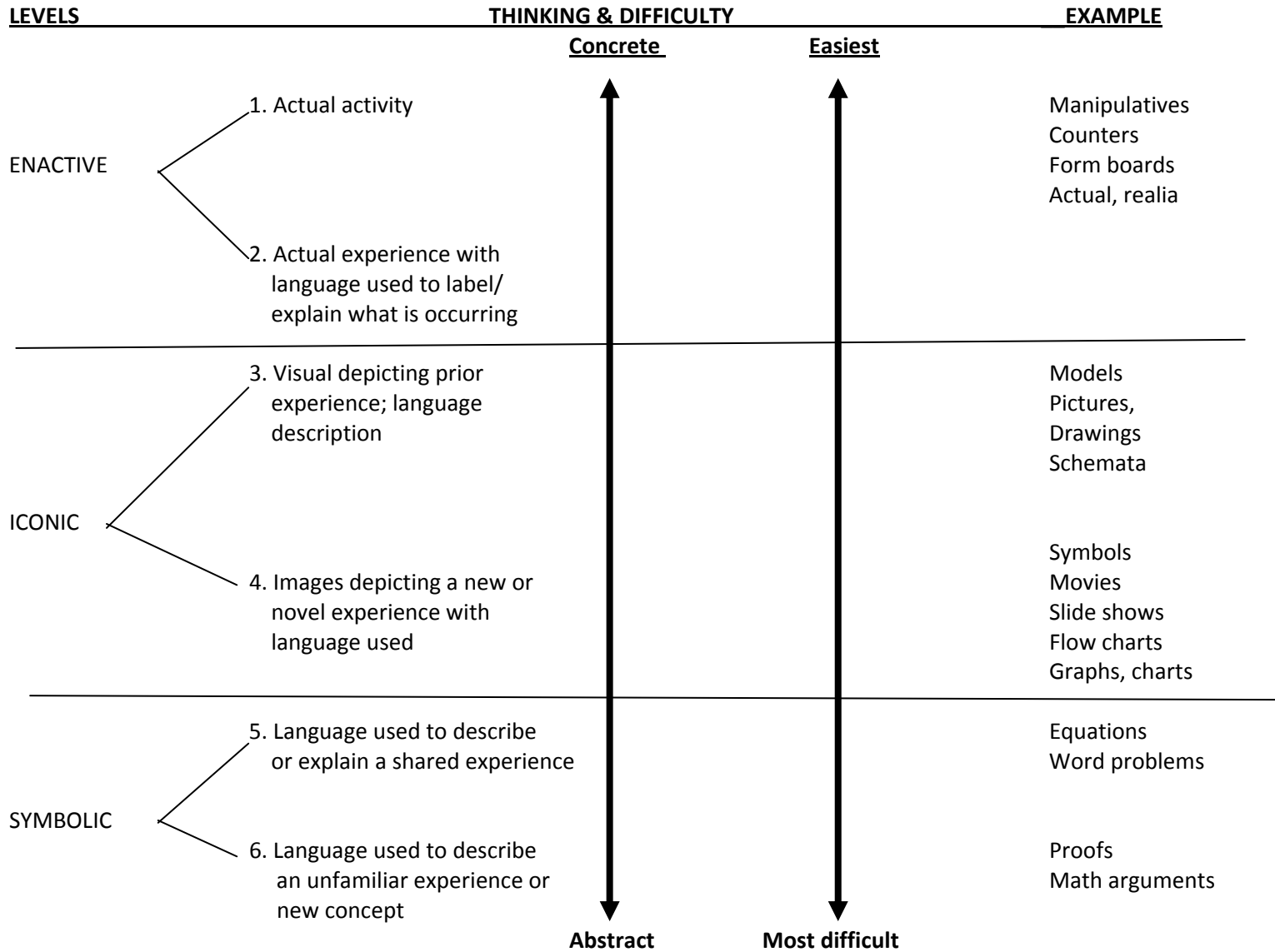
There are many well known strategies and instructional practices that help make lessons comprehensible and which set up the conditions for acquiring a new language. Making math comprehensible warrants contexts that are concrete, language-laden, interactive, active, and relevant. Among the strategies and practices recommended for ELL are:

1. thematic instruction
2. cooperative/collaborative group/paired work
3. use of projects
4. acceptance of a student's attempts at language

5. listening to a student's language without judgment as to its linguistic correctness
6. interpreting to the student's gestures and body language in signaling intent
7. preview of lessons in students first language (L_1) with the lesson following in the second language (L_2) and a review in L_1
8. use of materials in a student's native language
9. continuously checking for understanding
10. involving new language learners in class activities
11. providing assistance with the language of instruction
12. use of native language glossaries
13. focus on fewer subjects but each in more depth;
14. And, the overriding principle: *When the language is known, focus on the mathematic instruction; when the content is familiar, focus on the language*

Mathematics is a subject ripe with possibilities for new language learners. The use of words, expressions, sentences and equations to express quantity, chance and change are in many cases new even to those who speak the language. Mathematic instruction requires deliberate guidance and constant feedback to ensure achievement and progress in mathematics. To help the learning process, mathematics instruction also employs enactive experiences with concrete, hands-on manipulatives, realia, and forms to assist comprehension. In addition, iconic representations in graphs, pictures, charts, and other visuals are used to model real world events mathematical world. These instructional practices along with an appropriate introduction to and teaching of the language of mathematics can serve as the language learning laboratory for English language learners. The chart on the next page depicts the access uses for various types of classroom activities and language.

LEVELS OF DIFFICULTY AND ACCESS POINTS



The Transitional Bilingual Education Program in Washington State has identified 5 levels of English language proficiency for ELL students. Each level speaks to a range of English that progresses from a beginning level where little or no listening or speaking English proficiency is evident through three intervening levels where a student’s English progressively improves to a final transitional level of near native language fluency. At each level, different activities and instructional tools have been found to aid instruction. In the following chart, these are enumerated.

OPERANT PEDAGOGY AT 5 LANGUAGE PROFICIENCY LEVELS

<u>LEVEL</u>	<u>ACTIVITIES</u>	<u>INSTRUCTIONAL TOOLS</u>
<p><u>Beginning Level</u></p> <p>Primarily receptive language beginning to tie word/phrase with object or action</p>	<ul style="list-style-type: none"> • watch • recognizes • copy • recite • repeat • identify • label • matches • follow a pattern/- demonstration • associate • records • locates • copies • represents visually or graphically • memorizes common verbal expressions 	<ul style="list-style-type: none"> • manipulatives • real objects • cuisenaire rods • three-dimensional models • clocks • data tallies on graphs/charts • problem solving tools • number problems
<p><u>Advanced Beginning</u></p> <p>Mostly receptive. Actively making sense of situations and using telegraphic/speech in participating in lessons. Language is concrete and at times lacking in specificity. Can understand many directions. Students may use the math they know to learn English referents.</p>	<ul style="list-style-type: none"> • restates • describes • asks and responds to simple WH questions • matches pictures of varying quantities with math-related words or phrases • identifies • labels • simple math expressions • simple verbal and math expressions • labels steps in solving problems • draws 	<ul style="list-style-type: none"> • drawings/visuals • realia • objects • manipulatives • visuals • graphs • charts • math computation • numeric problem solving

<u>LEVEL</u>	<u>ACTIVITIES</u>	<u>INSTRUCTIONAL TOOLS</u>
	<ul style="list-style-type: none"> • creates charts and graphs • selects problem-solving tools • math computation • relies on visual support to solve problems 	
<p><u>Intermediate</u></p> <p>Language appears well developed but is unrefined. Definitions may be over-generalized and subtleties in meaning are missed. Iconic/visual/model representations help with meaning. Can work with assistance when descriptions of procedures are provided</p>	<ul style="list-style-type: none"> • language associated with different operations • tells • explains processes and algorithms • simple word problems • provides examples • pronounces math expressions accurately • begins to translate verbal to mathematic expressions and vice versa • suggests alternate routes to problem solving • explains apparent patterns presented visually • uses math words, phrases, symbols and illustrations • completes repeated math patterns of alternating figures described orally • follows directions • asks for clarification • describes math representations and operations from pictures of everyday objects and oral descriptions • compares/contrasts • shares real life applications of math lessons 	<ul style="list-style-type: none"> • word problems • numeric problems • graphs and charts • scales • measurements • physical models • problem-solving strategies • selects and explains problem-solving methods and tools to addresses everyday experiences • models • real life problems
<p><u>Advanced</u></p> <p>Syntax of the language is native-like but is still limited in breadth of lexical and semantic features. Precision is lacking. Definitions are superficial. Verging on</p>	<ul style="list-style-type: none"> • uses math sentences in appropriate sequence involving different operations • compares and contrasts using words, phrases, 	<ul style="list-style-type: none"> • word problems • calculations, computations • visuals, graphs, charts • compares/contrasts attributes of multi-

LEVEL	ACTIVITIES	INSTRUCTIONAL TOOLS
<p>abstractness in representing mathematic situations. Generates mathematical notation</p>	<p>symbols, or illustrations</p> <ul style="list-style-type: none"> • defines and solves problems by using logical reasoning and mathematical knowledge • defines mathematical terms accurately, succinctly, and precisely • discusses reasons for select use of problem solving strategies and procedures • selects and discusses problem solving methods • discusses conclusions and support them in familiar class situations • collects, displays, and interprets data for the class • shares possible outcomes of simple experiments • lists the steps used in solving problems • finds alternative ways of solving word and numeric problems • reflects on lessons learned • finds creative application of math lessons 	<p>dimensional figures</p> <ul style="list-style-type: none"> • word problems • calculations, computations • visuals, graphs, charts • use of and reasons for computations • summarization or prediction information • math sentences in appropriate sequences involving different operations • compares and contrasts using words, phrases, symbols, or illustrations
<p>Transitional</p> <p>Language is native-like and students communicate well. Every math lesson is an additional language lesson with new vocabulary introduced constantly, syntax expanded, and abstractness of meanings increased. Students use language to learn mathematics.</p>	<ul style="list-style-type: none"> • works independently after some explanation of expectations and modeling of operations • creates own math problems • uses math phrases or sentences accurately • predicts math patterns • oral descriptions • explains math problem solving • describes the use of visuals/ pictures in solving problems • uses math sentences to produce sequences for solving problems • explains math reasoning in 	<ul style="list-style-type: none"> • self created math problems • math phrases or sentences • math patterns • oral descriptions • numeric problem solving • visuals/ pictures in solving problems • math sentences to produce sequences for solving problems • real world problems • math phrases • simple math patterns • oral descriptions • sequences for solving problems

<u>LEVEL</u>	<u>ACTIVITIES</u>	<u>INSTRUCTIONAL TOOLS</u>
	<ul style="list-style-type: none"> selecting problem-solving strategies • accurately describes and applies function concepts and procedures to understand mathematical relationships • makes hypothesis, model situations, draw conclusions, and support claims • follows a line of reasoning in a mathematical argument • explains math problem solving • interpret mathematical situations • analyzes learning and suggests how they may improve in future lessons 	<ul style="list-style-type: none"> • steps to solving math problems • algorithms

ELL students along with native English speakers can learn to understand and use the language of mathematics. Teachers who intentionally attend to the language demands of mathematics will make instruction more comprehensible.

Math

English Development

Standards

MATH LANGUAGE PROFICIENCY LEVELS

The following descriptions enumerate the types of language arts capabilities indicative of K-2, 3-5, 6-8, and 9-12 students in listening/speaking, reading and writing at 5 levels of English language proficiency. Each grade level presumes that the students have the prerequisite knowledge of mathematics to handle instruction appropriate for their placement but lack the English competence to understand, speak, read, or write the language of instruction as well as might be expected of a native speaker of English. Obviously, it is recommended that instruction be adjusted for students whose mathematics background is either greater or lesser than that taught at their grade level placement. English language learners should receive instruction appropriate for their level of math performance and not have it dummed down due to their English facility. Because the behaviors are tied to the Washington State math curriculum within the grade clusters, these proficiency charts reflect the contention that English proficiency will improve as instruction is provided and mathematic understanding increases.

The Language Proficiency rubrics are not inclusive of all the math content taught at each grade cluster. They only present a sampling of the types of communicative abilities that may be manifested by students at each proficiency level. The descriptors do not constitute a checklist of all of the linguistic behaviors expected of students at each proficiency level but rather are a general indication of the types of language arts capabilities that may be revealed by ELL students involved in mathematics instruction. The math content to be covered at each grade level is included as a general guide to the contexts where language performance might be observed. It is clear that more specific content objectives will be present in individual lessons.

The charts that follow are designed for use by evaluators as holistic guides in forming a general impression of a student’s current level of language proficiency⁴. With holistic assessment, the evaluation rubric does not identify, list or account for all criteria that would signify separate levels of performance. Instead, a holistic rubric assigns a level by assessing language performance across multiple criteria as a whole. This type of performance assessment requires the evaluator to observe the student’s behavior or examine the product that is reflective of achievement, and then apply performance criteria in arriving at a professional judgment of the general level of language proficiency demonstrated.

For holistic assessment, the classroom environment must be one where students are actively involved with their peers in paired or group work and language abounds. In these classrooms, instruction moves beyond the tradition of teacher modeling, students practicing, and mastery testing to one in which students are engaged in using language to organize and consolidate their mathematical thinking, to communicate their mathematical thinking coherently and clearly to peers, teachers, and others; and to express mathematical ideas precisely⁵. In such classrooms, students generate and use language as they explore, represent, explain, investigate, formulate, predict, conjecture, discover, develop, solve, construct, and describe, justify, verify, and use mathematics.

⁴ For a specific manner of evaluating student performance within a mathematics lesson, primary trait analysis (analytic assessment) is recommended as it identifies criteria to be considered in scoring an assignment and/or response to a test item. In mathematics, such traits are usually nouns or phrases that are descriptive and can include a scale that evaluates the student’s numeric answer to a problem, step-by-step discussion of the strategy used in solving the problem, detailed consideration or examination of the proof offered, use of the language of mathematics, among others elements. The traits are used to establish a 3-point or 5-point scale (rubric) with explicit statements describing performance at each level and are tied directly to the problem under consideration. Student anchor papers are often used to establish consistency in evaluating student performance.

⁵ From the Five Process Standards in *Principles and Standards for School Mathematics* published by National Council of Teachers of Mathematics, 2000.

GRADES K-2

KINDERGARTEN

In kindergarten, students begin developing the concept of number by counting, representing and ordering, combining, sorting, and comparing sets of objects. They understand addition as putting sets together. In describing and identifying objects based on attributes and describing simple repeating patterns, students develop a beginning sense of geometry and algebra. They also develop an understanding of the relationship between data and concrete and pictorial representations of the data. They solve problems presented in their daily lives.

GRADE 1

In first grade, students count, sort, and compare sets, understanding the relative values of numbers. Students understand subtraction as separating, undoing addition, or comparing two numbers. They expand their understanding of number through application of basic addition and subtraction facts. Students read clock to the nearest hour, use nonstandard units to measure, and work with two-dimensional figures. First graders also develop their understanding of statistics by organizing data. They recognize and describe simple repeating and extending patterns to develop their algebraic sense. They begin to learn how to examine a question and select an appropriate problem solving strategy.

GRADE 2

In second grade, students expand their understanding of number to include three-digit numbers. They continue to gain proficiency in the basic addition and subtraction facts and expand concepts in measurement, using procedures to find measures of time, length, and weight. They expand their knowledge of the use of a number line and draw figures to match specific attributes. By interpreting and creating picture and bar graphs, students further develop their beginning understanding of statistics. Students also work with a variety of patterns and use symbols to describe numerical relations. Students increase their repertoire of problem solving strategies.

K-2	Listening/Speaking	Reading	Writing
Beginning	<ul style="list-style-type: none"> • Repeats number sequences • Associate quantity with sets of objects • Orders, sorts and compares sets of objects when provided a model along with oral instructions • Recites simple addition facts learned from use of manipulatives • Follows simple directions such as “adding or subtracting number quantities from a set • Identifies common shapes and manipulatives described/-labeled orally • Compares sets of objects using terms like more, less or the same 	<ul style="list-style-type: none"> • Recognizes number quantities • Matches pictures of various quantities of objects with math symbols • Understands numeric addition and subtraction math facts and matches them to pictures • Conducts addition computation to number problems • Associates common written names on worksheets with their pictures/-visuals • Replicates visual models of sets of objects • ‘Reads’ the hour hand of clocks 	<ul style="list-style-type: none"> • Begins to label quantities on worksheets • Can illustrate various whole number quantities (such as 1-100) • Begins to write addition math facts • Draws pictures representing simple addition math facts • Represents quantities and simple addition operations visually • Writes numbers with quantities they represent • Tallies physical qualities when provided a model (such as number of boys or girls; red, blue, white, or brown shoes; etc.)
Advanced Beginning	<ul style="list-style-type: none"> • Identify names of math objects based on attributes • Describes some attributes of objects such as color, number, etc. • Finds simple repeating patterns in shapes, number grids, etc. • Tells time to nearest hour • Labels the relationship between data and concrete and pictorial representations of data • Oral solve simple everyday addition problems • Follows steps in problem solving when accompanied by visual/graphic aids • Identifies math figures whose attributes are described (i.e. a four sided shape) 	<ul style="list-style-type: none"> • Identifies written expressions of mathematic operations (e.g. two plus two equals four) • Basic familiarity with mathematic terms such as those used with addition. • Reads math text haltingly identifying addition and subtraction numeric and operation terms • Reads simple graphs and charts and lists the data on them • Decodes attributes of common mathematic objects, manipulatives, etc. 	<ul style="list-style-type: none"> • Illustrates addition math problems • Creates graphs and charts from data obtained in class projects • Labels data on graphs and charts

K-2	Listening/Speaking	Reading	Writing
Intermediate	<ul style="list-style-type: none"> • Demonstrates understanding of relative values of numbers by counting, sorting and comparing sets • Describes subtraction as separating, undoing addition, or comparing two numbers • Lists basic addition and subtraction facts • Uses simple English to restates math operations of addition and subtraction from visual/pictorial representation. • Uses visuals, models, and everyday objects to state simple math operations • Examines a question and select an appropriate problem solving strategy • Relies on visual/pictures • Recognizes and describes simple repeating and extending patterns (beginning algebra sense) • Completes repeated math patterns of alternating figures described orally • Tells time to nearest hour and half hour • Uses nonstandard units to measure and work with two-dimensional figures • Orally generalizes from repeated mathematic data 	<ul style="list-style-type: none"> • Reads word problems and with guidance translates them to mathematic expressions • Able to identify useful and non-useful information in word problems • follows steps outlined in math texts for solving problems with assistance • Relies on visuals/pictorials to understand written textual material 	<ul style="list-style-type: none"> • Writes simple mathematic sentences • Describes the steps followed in solving a math problem • Writes words, symbols, or illustrations to represent whole numbers • Translates verbal expressions of simple addition problems to mathematic notation. • Creates simple addition and subtraction word problems • Completes addition operations • Writes simple addition formula for problem posed visually • Lists uses of whole numbers using words, phrases, symbols, or illustrations • Writes word problems • Writes generalizations from data or repeated mathematical patterns

K-2	Listening/Speaking	Reading	Writing
Advanced	<ul style="list-style-type: none"> • Discusses the use of nonstandard units to measure and work with two-dimensional figures • Describes the use of statistics for organizing data • Understands and explains addition and subtraction facts • Discusses concept of three-digit numbers • Uses and explains measurement concepts of time, length, and weight • Demonstrates a number line to draw figures to match specific attributes • Interpret pictures and bar graphs (statistics) • Explains the uses of a variety of patterns and symbols to describe numerical relations • Demonstrates the use problem solving strategies • Orally compose addition and subtraction word problems 	<ul style="list-style-type: none"> • Reads and comprehends the use of statistics for organizing data • Decodes a word problem and selects an appropriate strategy for solving the addition or subtraction problem. • Comprehends the uses of number lines when suggested in the mathematics text • Comprehends multiple problem solving strategies for addition and subtraction • Orders sentences sequentially outlining procedures that involve different operations 	<ul style="list-style-type: none"> • Uses simple and descriptive sentences • Uses statistics for organizing data • Constructs a number line to match specific attributes • Creates pictures and bar graphs (statistics) • Use symbols to describe numerical relations • Demonstrates the use of a variety of problem solving strategies • Write addition and subtraction word problems
Transitional	<ul style="list-style-type: none"> • Uses and explains measurement concepts of time, length, and weight • Discusses how a number line to draw figures to match specific attributes • Orally interpret and create pictures and bar graphs (statistics) • Use a variety of patterns and symbols to describe numerical relations • Lists sequential steps followed in addition and subtraction problem solving strategies 	<ul style="list-style-type: none"> • Adjusts reading rate as appropriate • Uses specialized vocabulary, uses multiple meaning words appropriately • Follows increasingly complex written directions for addition and subtraction operations • Reads and comprehends grade level mathematics text • Refers to the mathematics text glossary for definitions of new terms 	<ul style="list-style-type: none"> • Writes justification for the procedures followed in solving addition and subtraction problems • Uses specialized mathematics vocabulary when justifying solutions to addition and subtraction problems • Uses standard grammar and conventions when creating math story problems • • Writes own math problems using whole numbers in words, phrases, or

K-2	Listening/Speaking	Reading	Writing
	<ul style="list-style-type: none"> • Interpret and create pictures and bar graphs (statistics) to justify solutions to problems • Explains problem solving strategies • Applies content-related vocabulary in a variety of contexts and situations • Gives oral presentations explaining addition and subtraction problem solving strategies used 	<ul style="list-style-type: none"> • Interprets visuals and pictures in solving problems • Reads and comprehends mathematics text at grade level • Decodes simple math terms 	<p>sentences.</p> <ul style="list-style-type: none"> • Creates visuals and pictures to explain how problems were solved

GRADES 3-5

GRADE 3

In third grade, students develop their fluency with addition and subtraction, while beginning to understand multiplication as repeated addition and division as repeated subtraction and equal sharing. Students use standard units of measure for length, perimeter, time, money value, weight/mass, and temperature. They gain a broader understanding of geometry by using line segments and identifying properties of shapes. Algebraic sense grows through their understanding of equality and by identifying missing numbers in addition and subtraction expressions and equations. Students analyze, interpret and compare information in familiar situations.

GRADE 4

In fourth grade, students become proficient with multiplication and division of whole numbers, while developing an understanding of fractions and decimals. In measurement, they develop an understanding of area. The concept of probability as chance is developed and fourth graders continue to expand their understanding of statistics using graphing and measures of central tendency. Students refine their estimation skills for computation and measurement and develop an understanding of the relations between and among two-dimensional (plane) figures. They graph points in Quadrant 1 on a coordinate plane and extend and duplicate patterns. Students recognize geometric reflections and translations. They draw conclusions and support them in familiar situations.

GRADE 5

In fifth grade, students become proficient using non-negative rational numbers to solve problems. Fifth graders demonstrate an understanding of the concepts of divisibility including prime and composite numbers, fractions, and multiples. They apply procedures to measure a variety of geometric figures and collect, display, and interpret data. Students develop understanding of the likelihood of simple events and possible outcomes of simple experiments. They solve problems involving area and perimeter and further develop algebraic sense by using variables to write expressions and equations that represent familiar situations. They continue to check for reasonableness of answers. Students define problems by identifying questions to be answered when information is missing or extraneous and what is known and unknown in familiar situations.

3-5	Listening/Speaking	Reading	Writing
Beginning	<ul style="list-style-type: none"> • Follows teacher modeling in performing addition and subtraction • Counts • Recognizes numbers • Performs addition and subtraction under the guidance and modeling of teachers • Matches visual models of multiplication operations with their numeric expressions • Recognizes patterns on number charts 	<ul style="list-style-type: none"> • Matches multiple sets of objects with their numeric equivalents • Solves numeric addition and subtraction problems presented in the textbook • Decodes visuals in the math textbook showing multiplication as repeated addition 	<ul style="list-style-type: none"> • Demonstrates variable strategies for performing addition and subtraction operations • Develops visuals to show multiplication as repeated addition • Creates number charts
Advanced Beginning	<ul style="list-style-type: none"> • Labels units of length, perimeter, time, money value, weight/mass, and temperature on appropriate visuals/diagrams • Identifies units of length, perimeter, time, money value, weight/mass, and temperature on appropriate visuals/diagrams • Measures length, perimeter, time, money value, weight/mass, and temperature on concrete and visual representations of objects • Use line segments and identifies properties of shapes (geometry) • Expresses an understanding of 'equality' when solving problems 	<ul style="list-style-type: none"> • Identifies missing numbers in addition and subtraction expressions and equations • Analyzes, interprets and compare additive, subtractive, and multiplicative information presented visually in the math text and translates the information into mathematic expressions 	<ul style="list-style-type: none"> • Writes simple mathematic expressions to reflect addition, subtraction, and multiplication operations. • Creates visuals to represent addition, subtraction and multiplication operations
Intermediate	<ul style="list-style-type: none"> • Explains the procedures employed in the multiplication and division of whole numbers • Identifies fractions and decimals and 	<ul style="list-style-type: none"> • Decodes grade level math textbooks with difficulty and comprehends content only with assistance of an instructor 	<ul style="list-style-type: none"> • Lists the steps followed in solving multiplication and division problems • Creates visuals to represent fractions and decimals

3-5	Listening/Speaking	Reading	Writing
Intermediate-Continued	<ul style="list-style-type: none"> • matches them with visual representations • Explains how decimals and percentages are two additional methods for representing fractions • Discusses the concept of equivalent fractions • Reports on the measurement of areas of physical objects. • Distinguishes between chance and the concept of probability • Uses graphs and other data organizers to explain central tendency • Verbally estimates while solving computation and measurement problems • Points out the relationships between and among two-dimensional (plane) figures • Lists how to graph points in Quadrant 1 on a coordinate plane in order to extend and duplicate patterns • Recognizes and identifies geometric reflections and translations 	<ul style="list-style-type: none"> • Able to comprehend and employ algorithms presented in the text • Translates set models of fractions into numeric expressions • Reads graphs; coordinates, points, line, etc. 	<ul style="list-style-type: none"> • Records the measurement of familiar objects found in the classroom • Uses fraction symbols such as numerator and denominator • Visually depicts which fraction in a pair is greater • Records procedures for working with fractions, decimals, and percentages • Journals generalizations from working with fractions, decimals, and percentages • Translates numeric fraction, decimal, and percentage quantities into verbal expressions • Generates simple word problems involving fractions, decimals, and percentages
Advanced	<ul style="list-style-type: none"> • Discusses conclusions and support them in familiar class situations • Explains the use non-negative rational numbers in solve problems • Enumerates situations where concepts of divisibility including prime and composite numbers, fractions, and multiples apply • Describes measurements of a variety of geometric figures 	<ul style="list-style-type: none"> • Reads and understands grade level material with little assistance • Explain conclusions discussed in the grade level math text • Follow text suggestions for solving problems containing non-negative rational numbers • Compare and contrast different problem solving strategies • Uses estimation in beginning to solve 	<ul style="list-style-type: none"> • Writes word problems • Translates word problems into mathematic expressions • Construct a model from measurements • Summarize data from a graph or chart and interpret it in a report

3-5	Listening/Speaking	Reading	Writing
Advanced Continued	<ul style="list-style-type: none"> • Collects, displays, and interprets data for the class • Shares possible outcomes of simple experiments • List the steps used in solving problems involving area and perimeter 	<p>math problems</p> <ul style="list-style-type: none"> • Follows conversion procedures for fractions, decimals, percentages, 	
Transitional	<ul style="list-style-type: none"> • Explains how to solve problems involving area and perimeter • Uses accurate mathematical notation in expressing equations that represent familiar situations (algebra) • Explains reasonableness of answers • Defines problems by identifying questions to be answered when information is missing or extraneous and what is known and unknown in familiar situations. 	<ul style="list-style-type: none"> • Reads and comprehends math textbooks at grade level • Independently able to decode a word problem and translate it into a mathematical expression to be solved • Interprets and uses algorithms • Determines what information is missing or extraneous in written problems • Translates word problems into math expressions to be solved 	<ul style="list-style-type: none"> • Writes expressions and equations using variables to represent familiar situations • Writes to explain reasonableness of answers • Translates math expressions into word expressions • Composes real world word problems

GRADES 6-8

GRADE 6

In sixth grade, students expand their concept of number to include integers, fractions and decimals. They examine the concepts of ratio and volume, as well as collect, analyze, display, and interpret data, using a variety of graphical and statistical methods. They find all possible outcomes of events for simple experiments. They analyze numerical and geometric patterns. Students also develop an understanding of algebraic terms and solve algebraic equations in one variable.

GRADE 7

In seventh grade, students begin to work with the inverse property and the concept of direct proportion in computations with decimals, fractions, and integers. Fluent use of strategies for all operations on non-negative rational numbers is expected of students. They locate points in any of the four quadrants on a grid and translate linear relations in table, graph, and equation forms. Students extend their understanding of complimentary and mutually exclusive events. Algebraic sense also develops as students solve two-step equations in one variable. Students gather, organize, and share mathematical information for a given purpose. They use mathematical language to present information.

GRADE 8

In eighth grade, students are proficient in computation with rational numbers and use ratio, percent and direct proportion to solve a variety of problems. They understand the need for precision when measuring and use derived units of measure. Students understand the concept of distance and the relation between distance and the Pythagorean Theorem. They recognize three-dimensional figures represented in two-dimensional drawings and apply transformations to geometric figures in the coordinate plane. Eighth graders find probability of compound events and analyze bivariate data sets. They also understand recursive forms of linear and exponential relations and solve two-step equations and inequalities. Students solve problems by selecting and using appropriate concepts and procedures. They determine whether the solutions are viable and mathematically correct.

6-8	Listening/Speaking	Reading	Writing
Beginning	<ul style="list-style-type: none"> • Repeats names of integers, fractions and decimals • concept of number includes • Matches integers, fractions and decimals with visual or physical representation • Follows oral directions accompanying demonstration in solving math problems involving integers, fractions, and decimals • Follows physical demonstration and oral description in determining ratios of two objects and volume of objects 	<ul style="list-style-type: none"> • Reads and solves numeric problems involving integers, fractions and decimals • Matches data on graphs and charts to items they represent. • Records data on charts from hands-on activities • Recreate numeric and geometric patterns • Evaluates (solves) simple numeric algebraic problems in text or workbooks • Repeats math expressions of numeric problems • Identifies operations in mathematic problems 	<ul style="list-style-type: none"> • Visually uses number lines depicts concepts of negative and positive integers • Creates graphs or charts to depict fractions and decimals • Translates visual depictions of different sized objects to their mathematic ratio expression • Translates visual depictions of different objects to their mathematic ratio expression • Shows steps in solving numeric problems
Advanced Beginning	<ul style="list-style-type: none"> • Matches and/or labels data displayed graphically • Matches statistical notation with their referent • Collects, analyzes, displays, and labels data, using a variety of graphical and statistical methods • Finds all possible outcomes of events for simple experiments • Labels numerical and geometric patterns • Knows algebraic terms and is able to label variables in 	<ul style="list-style-type: none"> • Identifies data presented graphically • Reads and solves numeric algebra problems involving one variable • Employs algorithms presented visually in the textbook in solving numeric operations 	<ul style="list-style-type: none"> • Displays collected data using graphs and fundamental statistical methods • Visually displays and labels numeric and geometric patterns • Correctly employs statistical and graphic means to represent data

6-8	Listening/Speaking	Reading	Writing
	simple algebraic equations		
Intermediate	<ul style="list-style-type: none"> • Inverse property and the concept of direct proportion in computations with decimals, fractions, and integers • Fluent use and explanation of strategies for all operations on non-negative rational numbers • Locates points in any of the four quadrants on a grid • Translates linear relations in table, graph, and equation forms into mathematic notation • Comprehends a discussion of the contrast between complimentary and mutually exclusive events when presented visually • Solve two-step equations in one variable (algebra) and orally labels the operations employed 	<ul style="list-style-type: none"> • Reads and solves simple word problems with guidance • Accurately reads decimals, fractions, and integers using mathematical notation • Comprehends descriptions on the construction and use of tables, graphs, and simple equations • Solves numeric problems involving inverse property and the concept of direct proportion 	<ul style="list-style-type: none"> • Writes to explain the difference between complementary and mutually exclusive events visually and in writing • Outlines the steps in solving algebra equations containing one variable • Shows visually and numerically how to solve problems of inverse property and the concept of direct proportion using decimals, fractions, and integers
Advanced	<ul style="list-style-type: none"> • Gathers, organizes, and shares mathematical information for a given purpose • Uses mathematical language to present information • Explains the use of ratio, percent and direct proportion in solving a variety of 	<ul style="list-style-type: none"> • Able to read and solve word problems from the textbook • Comprehends data presented in visuals, graphs, and charts • Reads and understands how to use geometric figures and other line segments 	<ul style="list-style-type: none"> • Independently maintains a math journal • Lists and explains steps followed in solving problems • Compares and contrasts strategies using words, phrases, symbols, and/or illustrations • Writes math sentences in appropriate sequence involving different operations

6-8	Listening/Speaking	Reading	Writing
	problems <ul style="list-style-type: none"> • Precision when measuring • Uses derived units of measure • Discusses the concept of distance with peers involved in group work 		
Transitional	<ul style="list-style-type: none"> • Discusses relation between distance and the Pythagorean Theorem • Describes three-dimensional figures represented in two-dimensional drawings • Orally applies transformations to geometric figures in the coordinate plane • Discusses probability of compound events • Analyzes bivariate data sets • Explains linear and exponential relations • Enumerates how to solve two-step equations and inequalities • Orally solves problems by selecting and using appropriate concepts and procedures • Explains whether solutions are viable and mathematically correct. 	<ul style="list-style-type: none"> • Reads grade-level text with good comprehension • Comprehends and is able to solve word problems • Accurately interprets data organized and visually displayed on graphs and charts • Reinterprets and summarizes textbook explanations using more communicative and easier to understand English • Comprehends descriptions of bivariate data sets • Translates written descriptions of linear and exponential relations into mathematic terms 	<ul style="list-style-type: none"> • Solves and explains word problems • Able to explain steps and reasoning in solving problems • Verbally discusses how to use geometric figures and other line segments • Develops a narrative explaining data reported on graphs and charts • Writes an explanation of the relationship between distance and the Pythagorean Theorem • Describes three-dimensional figures represented in two-dimensional drawings • Creates and solves word problems • Develops an explanation to why solutions to problems are viable and mathematically correct

GRADES 9-12

Grades 9/10

In ninth and tenth grades, students will be proficient with operations on rational numbers represented in all forms including scientific notation. Students analyze effects of changes in dimensions of two-dimensional and three-dimensional figures and apply formulas to measurement. Students use a variety of methods and formulas to find area, volume, the slope of a line, and the distance between points on a coordinate grid. They apply multiple transformations to points in the plane and can apply conditional probability in situations. Students write equations for lines. Students analyze, compare, and integrate mathematical information from multiple sources. They also justify results using inductive or deductive reasoning.

Grades 11/12

In the eleventh and twelfth grades, students will be able to use logical reasoning and mathematical knowledge to define and solve problems. Students will apply strategies and use procedures related to real numbers. Students will accurately describe and apply function concepts and procedures to understand mathematical relationships. They also make hypothesis, model situations, draw conclusions, and support claims using geometric concepts and procedures. They continue to maintain and expand algebraic skills.

9-12	Listening/Speaking	Reading	Writing
Beginning	<ul style="list-style-type: none">• Records and labels outcomes of probability events• Repeat names of exponents• Label each factor in products expressed in scientific notation• Follow oral directions and examples provided visually in solving problems that involve operations on rational numbers	<ul style="list-style-type: none">• Read and solve numeric problems involving operations on rational numbers in all forms including scientific notation	<ul style="list-style-type: none">• Visually represent problems involving rational numbers• Converts fractions to decimal numbers• Copies and solves number problems involving rational numbers
Advanced Beginning	<ul style="list-style-type: none">• Label each factor in products expressed in scientific notation• Labels effects of changes in dimensions of two-dimensional and three-	<ul style="list-style-type: none">• Able to follow a schematic flow chart describing methods and formulas to find area, volume, the slope of a line, and the distance between points on a coordinate grid• Recreates two and three	<ul style="list-style-type: none">• Collects definitions of important terms and equations• Create two and three models to demonstrate effects of dimensional changes• Matches pictures of varying quantities with math-related words or phrases

9-12	Listening/Speaking	Reading	Writing
	<p>dimensional figures</p> <ul style="list-style-type: none"> • Applies formulas to measurement tasks • Identifies and uses a variety of methods and formulas to find area, volume, the slope of a line, and the distance between points on a coordinate grid • Using charts, graphs, or other visuals describes effects of changes in dimensions of two-dimensional and three-dimensional figures • Selects formulas to apply to measurement tasks • Uses simple math expressions • Asks and responds to simple WH questions 	<p>dimensional models patterned after those found in grade level math textbooks</p>	
Intermediate	<ul style="list-style-type: none"> • Provides examples of the application of multiple transformations to points in the plane • Explains how to apply conditional probability in situations • Equations for lines • Orally analyzes, compares, and integrates mathematical information from multiple sources • Describes math representations and operations from pictures of 	<ul style="list-style-type: none"> • Replicates inductive and deductive reasoning patterns from written text in explaining use of problem solving procedures • With assistance translates mathematical expressions and problems to verbal expressions and vice versa • Accuracy in reading mathematic notation 	<ul style="list-style-type: none"> • Keeps a math journal of insights, learned concepts, definitions, and examples of important procedures • Analyzes, compares, and integrates mathematical information from multiple sources • Translates mathematical expressions and problems to verbal expressions and vice versa • Outlines procedural steps followed in solving a math problem

9-12	Listening/Speaking	Reading	Writing
	everyday objects and oral descriptions		
Advanced	<ul style="list-style-type: none"> • Defines and solves problems by using logical reasoning and mathematical knowledge • Defines mathematical terms accurately, succinctly, and precisely • Discusses reasons for select use of problem solving strategies and procedure related to real numbers • Selects and discusses problem solving methods 	<ul style="list-style-type: none"> • Reads and comprehends grade level math textbooks • Independently able to convert word problems into mathematic expressions • Utilizes internet math resources 	<ul style="list-style-type: none"> • Uses math sentences in appropriate sequences involving different operations • Compares and contrasts problem solving and reasoning strategies accurately using math words, phrases, symbols, or illustrations • Writes reports using mathematic evidence to support claims, generalizations, and conclusions • Writes an-easy-to understand newspaper article incorporating data from complicated mathematic reasoning. • Justify results using inductive or deductive reasoning

Appendix

APPENDIX I

Consider the language of mathematics encountered by first graders. In the following, what vocabulary and language patterns (math sentences) would students need to understand in order to solve the problem involving the addition of squares? What language would these students need in order to communicate the strategy(ies) they used in completing this addition task?

Add the following squares

$$\square \square \square \square + \square \square \square = \underline{\quad}$$

TEACHER'S LANGUAGE

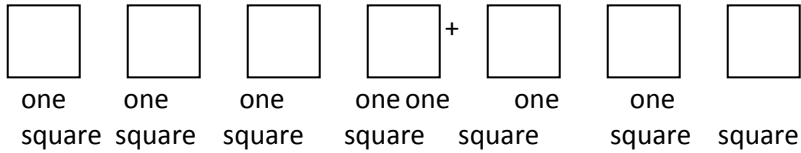
The teacher, in assigning and guiding this exercise, uses language, the language of mathematics, to provide direction and to share examples of how to enact the process of addition. This language might include any or all of the following phrases and questions:

- ✓ How many total squares are there?
- ✓ Count the squares on the left side of the + sign.
- ✓ Count the squares on the right side of the + sign.
- ✓ What does the "+" ask you to do?
- ✓ Add the number of squares
- ✓ How many are there altogether?
- ✓ How many in all?
- ✓ How many squares are on the left side of the + sign? How many on the right side?
- ✓ Add the two numbers.
- ✓ What is the answer when you add the numbers on each side together?
- ✓ What does the "=" mean?
- ✓ How much is 4 and 3?
- ✓ What is the sum of 4 and 3?
- ✓ What is 4 plus 3?
- ✓ What number do you get when you add the 4 and 3 together?
- ✓ The 4 is increased by how many?
- ✓ How many did you add to the 4?
- ✓ Four squares and three more are....
- ✓ Four plus three equals....
- ✓ What is the answer?
- ✓ Where do you write your answer?
- ✓ What did you do when you saw the "+" sign?
- ✓ Tell me how you got your answer.
- ✓ What did you do to solve this problem?
- ✓ Is there another way to solve this problem?

STUDENT'S UNDERSTANDING OF MATHEMATIC EXPRESSIONS

In addition to understanding the teacher's language, the student needs to be familiar with various vocabulary items and the manner of expressing them:

- ✓ a. number sequence: 1, 2, 3, 4, 5, 6, 7 and their verbal notations one, two, three, etc.
- ✓ b. number association:



ADDITION OPERATION:

- ✓ number quantity: $1 + 1 + 1 + 1$ is 4 squares
4 squares + 1 square + 1 square + 1 square = a number
4 squares plus 3 squares = 7 squares
- ✓ + : plus, added to, and, sum of, combined with, increased by
- ✓ =: equals, is, is the same as
- ✓ $4 + 3 =$ a number

Expressions:

- ✓ $4 + 3 = 7$
- ✓ 4 squares plus 3 squares equals 7 squares.
- ✓ The sum of 4 squares plus 3 squares is 7.
- ✓ When I add three squares to four squares, the result is seven squares.
- ✓ Four squares increased by three squares equals seven squares.
- ✓ Four squares combined with three squares results in a total of seven squares.

Processes:

- ✓ Problem – 4 squares plus 3 squares = a quantity of squares
- ✓ The numbers to the left of the = sign are equivalent (the same total value) as the number to right of the equal sign.
- ✓ Use of patterns:
Two fours is eight. Three is one less than four, so eight minus one is seven.
Counting up, I have four squares. One more square is five. Another makes the quantity six.
And the final one added makes seven.

COMMON PROBLEMS ENCOUNTERED

First grade children at times can become confused when using mathematical notation and mathematical verbal expressions.

Common Errors:

$4 + 3$ “becomes” 7

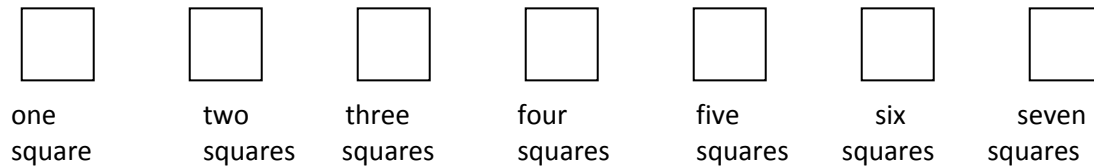
Four squares plus three squares
“are” seven squares

Correct notation

$4 + 3$ is 7

Four squares plus three squares “is” seven squares
(The sum of the quantity of four plus three is seven.)

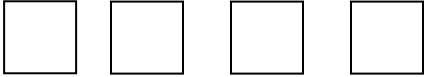

At times in counting objects, students might assume that each square is a summation of those that preceded it in the count. They may say the following:



Explanation: Each is one square. To add, the number quantity increases with each square and is stated: The sum of the squares is seven. The fourth individual square in the sequence is not four squares, it is **one more** square. A student more accurately might say:

One square plus one square is two squares. Two squares plus one additional square is three squares. Three squares plus one more square is four squares. etc.

Or, it might be depicted as follows:

	+		= ___
1 square + 1 square + 1 square + 1 square		1 square + 1 square + 1 square is three squares	
4 squares	Plus	Three squares	Equals

The sum of four squares and three squares is seven squares.
 When three squares are added to four squares, the result is seven squares.
 Four squares plus three squares equals seven squares.

Another error is found in “4 + 3 = a number seven” (suggesting that this can one of many numbers).

Explanation: The expression should be 4 + 3 = (equals) the number seven (indicating there is only one such answer); or 4+3 is 7. Another way of stating this is: The number 3 added to the number 4 is 7.

By this point in mathematics education, students have memorized a number sequence beginning with one and ending at a much larger number. They have learned to associate the number word with an object or a set of objects. They have practiced math facts of addition and have experienced using addition strategies. Perhaps, they also have begun to learn subtraction facts. And, they have worked with the concepts of less, more, greater, fewer, and the idea of sets. Their language of mathematics now should include simple declarative sentences. Remember, this language was intended for 6 or 7 year old children

Appendix II

WASL

Mathematic

Release Items

Mathematic WASL Sample Questions

The following math problems are taken from the 2006 WASL release items. To assist student's access to the math curriculum, math and ESL/bilingual teachers reviewed the English language demands for each of the math problems and selected vocabulary that could be confusing. They also identified background knowledge that a student would need to solve the problem. This process provides an avenue for teachers to not only focus on the solution to a problem but to target the language a student needs in order to understand and solve mathematical problems.

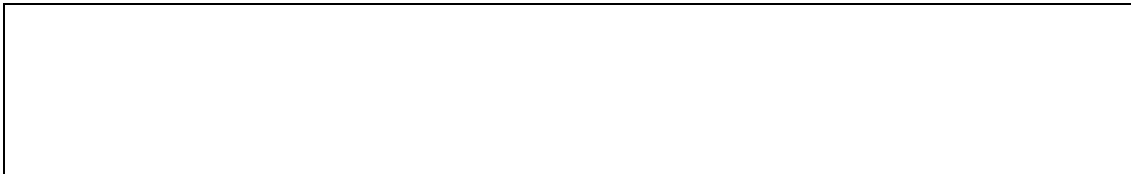
2006 Mathematics Released Items – Samples

1. Tim is making a sign. He wants a figure (closed shape) with 5 sides. One side of the figure needs to be 3 inches long.

Draw a figure for the sign.

- Draw a figure with 5 sides.
- Make one side 3 inches long.
- Label the side that is 3 inches long.

You must use a ruler or straightedge.



ESSENTIAL LANGUAGE	
Vocabulary	Instruction
closed shape	Draw a closed figure
sides	Label the side
figure	Draw a side 3 inches long
measure	Measure with ruler to see if it's 3 inches long
long	
label	
straight edge	
sign	

Background Knowledge – information students should have to solve problem.

- How to draw a straight line to a certain measure
- How to draw a closed figure

- How to label a side to a certain measure

2006 Mathematics Released Items – Samples

3. Bonita had 12 comic books. Anan gave Bonita more comic books. Now Bonita has 20 comic books.

- Write an addition equation that can be used to find how many comic books Alan gave Bonita.

Numbers and Symbols

$$20 + = 12$$

- Write a subtraction equation that can be used to find how many comic books Alan gave Bonita.

Numbers and Symbols

$$20 - = 12$$

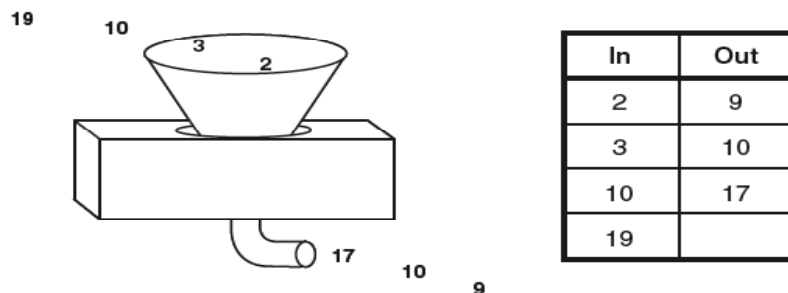
Vocabulary	Instruction
Equation Comic book	Write an addition equation Write a subtraction equation How many did you start with? How many did you get How many do you have now?

Background Knowledge – information students should have to solve problem.

- Use fact family
- Addition
- Subtraction
- Equation
- Fact family
- Unknown (variable)
- Symbols: +; -; =; unknown number _ or a box

2006 Mathematics Released Items

2 The diagram below is a function machine.



What is the next number that will come out of the function machine?

After putting in 19, the number that comes out of the function machine is _____.

What is the rule for this function machine?

ESSENTIAL LANGUAGE

Vocabulary	Instruction
diagram function machine in out rule/pattern below table	How do you get from 2 to 9? How do you get from 3 to 10? What is the pattern? What is the next number in this pattern? What is the rule for this pattern?

Background Knowledge – information students should have to solve problem.






- Know that a function machine follows a rule/pattern
- Know that you put a number into the function machine and after the operation another number comes out
- Addition
- Familiarity with a table

2006 Mathematics Released Items

- 3 Shivani's class was planning a luncheon and decided to set up a salad bar. They took a survey to find their classmates' favorite salad toppings.

This is how the class voted:

Favorite Salad Toppings

Broccoli		3
Carrots		5
Cheese		4
Cucumbers		6
Tomatoes		4

ESSENTIAL LANGUAGE

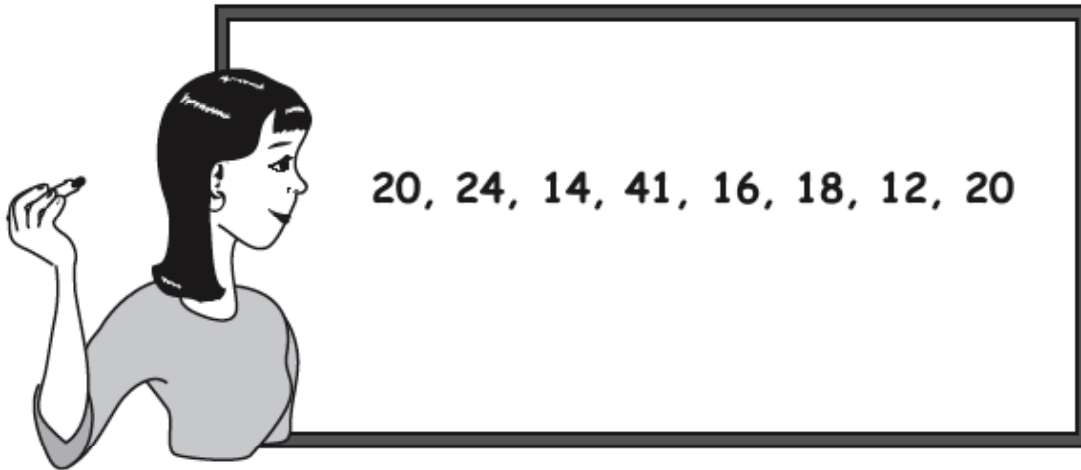
Vocabulary	Instruction
luncheon salad bar salad toppings vote survey 1 or 2 data table scale bar graph categories label title horizontal axis vertical axis interval	Use this data to make a bar graph Include . . . How do you take a survey? Write a title appropriate scale with intervals of consistent scale label for the horizontal axis label for the vertical axis

Background Knowledge – information students should have to solve problem.

- Parts of a graph
- Vocabulary related to making a graph
- How to translate data from a table to a graph

- 1 The students in Mrs. Middleton's class learned that reducing the number of times they open their refrigerator doors saves energy. They kept track of how many times their families opened their refrigerator doors in one evening.

Mrs. Middleton wrote their data on the board and asked the students to find the median (middle number).



What is the median of the class data?

Show your work using words, numbers, or pictures.

What is the median of the class data? _____

ESSENTIAL LANGUAGE

Vocabulary	Instruction
Reduce Keep track Data Median (middle number) Mode Mean Least greatest	Show your work using words, numbers, or pictures What is the median of . . . data? Put the numbers in order from least to greatest

Background Knowledge

- Know the difference between mean, median, and mode
- Understand the rule of operation with even number in the collection

Expressions:

Show your work using words, numbers, or pictures
What is the median of . . . data?
Put the numbers in order from least to greatest

GRADE 6

1. The Eatenburg Middle School principal needs help writing a survey question to collect information from the students in the school. She wants to find out if the students are willing to buy healthier, but more expensive lunch items.

Which question should the principal put on the survey to get the information she needs?

- A. "How many school lunches do you usually buy each week?"
- B. "How much money do you spend on school lunches each week?"
- C. "How many times each week do you buy dessert with your school lunch?"
- D. "How much money would you be willing to spend for healthier school lunches?"

ESSENTIAL LANGUAGE

Vocabulary	Instruction
Survey question Collect information Find out healthier More expensive	Survey survey question collect

Background Knowledge – information students should have to solve problem.

- many
- more
- money
- times
- each week

Mathematics

- 2 Mr. Kleine plays a game called “Use my Rule” in his class. He says the following expressions are called zendo:

$$3 \rightarrow \boxed{13}, 5 \rightarrow \boxed{19}, \text{ and } 1 \rightarrow \boxed{7}$$

The rule for a zendo is to multiply the starting number by 3 then add 4, or $3t + 4$.

Mr. Kleine asks Sherry to help him come up with examples for a new expression called crardird that follows the rule, $t - 2$.

Which of the following groups contain all correct examples of crardirds?

Crardird rule: $t - 2$

A. $2 \rightarrow \boxed{4}$

$4 \rightarrow \boxed{6}$

$6 \rightarrow \boxed{8}$

B. $2 \rightarrow \boxed{0}$

$4 \rightarrow \boxed{2}$

$6 \rightarrow \boxed{5}$

C. $3 \rightarrow \boxed{1}$

$5 \rightarrow \boxed{3}$

$7 \rightarrow \boxed{5}$

D. $3 \rightarrow \boxed{2}$

$5 \rightarrow \boxed{4}$

$7 \rightarrow \boxed{6}$

ESSENTIAL LANGUAGE

Vocabulary	Instruction
expression $3t + 4$ multiply add rule examples	expression (add/subtract/multiply/divide) rule examples

Background Knowledge – information students should have to solve problem.

- rules
- expressions
- patterns
- groups

Mathematics

- 3 Ashur had a collection of toy cars. The collection consisted of red, blue, green, and yellow toy cars. Ashur had twice as many yellow toy cars as all the other colors combined. He had 120 toy cars in his collection.

How many yellow toy cars did Ashur have in his collection?

Show your work using words, numbers, and/or pictures.

<p>How many yellow toy cars did Ashur have in his collection? _____</p>

ESSENTIAL LANGUAGE

Vocabulary	Instruction
collection consisted combined twice as many	combined twice as many consisted

Background Knowledge – information students should have to solve problem.

- sets/collections
- categories
- more
- less
- many

Mathematics **Sample**

Kirta filled 27 cartons of eggs every 3 minutes.

Which ratio is equivalent to 27 cartons every 3 minutes?

- A. 36 cartons every 12 minutes
- B. 54 cartons every 9 minutes
- C. 135 cartons every 15 minutes
- D. 216 cartons every 27 minutes

ESSENTIAL LANGUAGE	
<u>Vocabulary</u>	<u>Instruction</u>
every (per) ratio equivalent cartons filled	every/per ratio equivalent

Background Knowledge – information students should have to solve problem.

- Know the number of minutes in an hour
- Understanding of equivalent
- Understanding of ratio

OSPI Mathematic Glossary

Absolute value

The numerical value of a number without regard to its sign; the distance from 0 to a point on the number line ($| |$ means absolute value).

Examples: $| 3 | = 3$ and $| -3 | = 3$; $| 9 | = 9$ and $| -9 | = 9$; $| 0 | = 0$.

Acute angle

An angle which measures less than 90 degrees and greater than 0 degrees.

Acute triangle

A triangle with three acute angles.

Addend

Any number that is added; addend + addend = sum.

Example: in $3 + 4 = 7$, the 3 and 4 are addends.

Addition

An operation joining two or more sets where the result is the whole.

Adjacent angles

Angles in the same plane that have a common side and a common vertex, but whose interiors do not intersect.

Example:



Algorithm

A step-by-step method for computing.

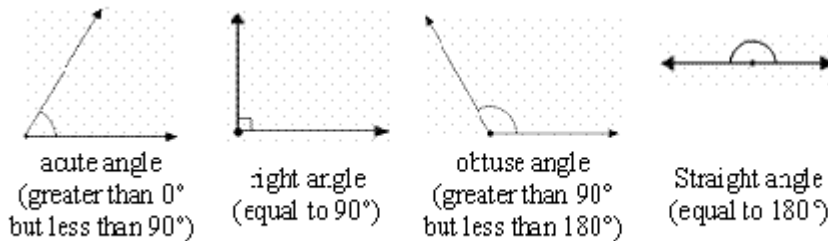
Analyze

To breakdown a whole into component parts so that it may be more easily understood.

Angle

Two rays that share an endpoint; classified according to the number of degrees of its measure.

Examples:



Approximate

To obtain a number close to an exact amount.

Area

The area of a flat or plane figure is the number of unit squares that can be contained within it. The unit square is usually some standard unit, like a square meter, a square foot, or a square inch.

Argument

A reason or reasons offered for or against something; suggests the use of logic and facts to support or refute a statement or idea.

Arithmetic progression or sequence

A list of numbers, called terms, in which the difference between any two adjacent terms is the same. The first number in the list is called the initial value.

Example: The list 1, 3, 5, 7, ... is an arithmetic sequence because the difference between any two adjacent numbers is 2. That difference is called the common difference.

Associative property of addition

The sum stays the same when the grouping of addends is changed.

Examples:

$$(a + b) + c = a + (b + c)$$

$$(30 + 4) + 20 = 30 + (4 + 20)$$

Associative property of multiplication

The product stays the same when the grouping of factors is changed.

Examples:

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$$

Attribute

A characteristic or distinctive feature.

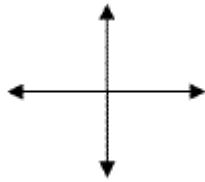
Average

A measure of central tendency; generally, average will imply arithmetic average, which could be the mean, median, or mode.

Axes

Perpendicular lines used as reference lines in a coordinate system or graph; traditionally, the horizontal axis represents the independent variable and the vertical axis the dependent variable.

Example:



Axiom

A self-evident and generally accepted statement.

Example: Two points determine exactly one line.

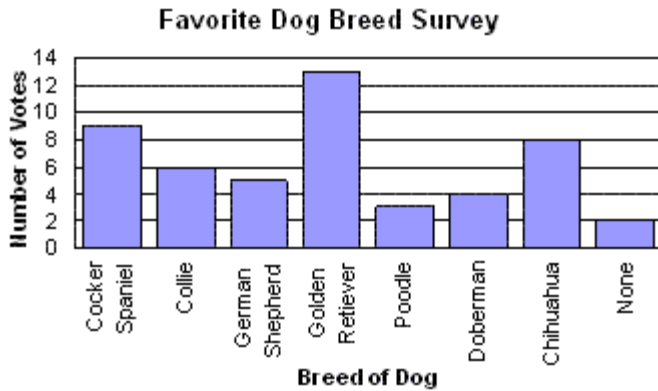
Axis

See x-axis and y-axis. The plural of "axis" is "axes".

Bar graph

A graph that uses the length of solid bars to represent numbers and compare data.

Example:



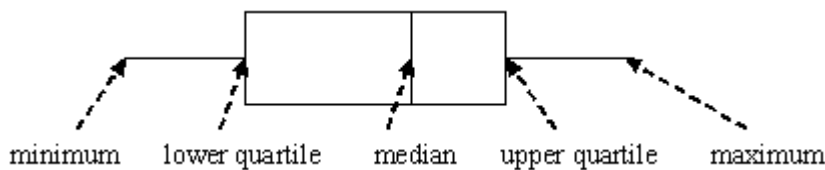
Bivariate data

Data involving two variables, such as height and weight, or amount of smoking and a measure of health; often graphed in a scatter plot.

Box-and-whisker plot

A graph which displays the following five points from a data set- the minimum value, the lower quartile (25th percentile), the median, the upper quartile (75th percentile), and the maximum value.

Example:



Capacity

Volume and capacity are both terms for the measures of the "size" of three-dimensional regions. Standard units of volume are expressed in terms of length units, such as, cubic centimeters. Capacity units are generally applied to liquids or the containers that hold liquids. Standard capacity units include quarts and gallons, liters and milliliters.

Cardinal number

Number that designates how many objects, or the number of units in the set; answers the question, "How many...?".

Example: There are 28 students in the room. The cardinality or cardinal number is 28.

Central tendency

A single number that describes all the numbers in a set. Three common measures of central tendency are mean, median, and mode.

Example: For the set of numbers 95, 86, and 83, the mean is 88.

Certain event

An event that will definitely happen. A certain event has a probability of 1.

Characteristic

A distinguishing element.

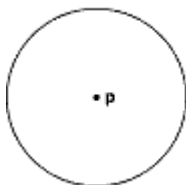
Chart

A method of displaying information in the form of a graph or table.

Circle

A set of points in a plane that are all the same distance from a given point.

Example: Circle P is drawn below.



Circle graph

Sometimes called a pie chart, a way of representing data that shows the fractional part or percent of an overall set as a corresponding part of a circle.

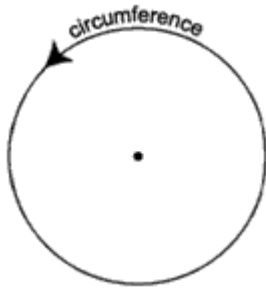
Example:



Circumference

The boundary, or perimeter, of a circle; also, the length of the perimeter of a circle.

Example:



Closure property

A set of numbers is said to be closed under an operation if the result of performing the operation on any two numbers in the set produces a number in the set.

Cluster

In terms of statistics, a relatively large number of data closely grouped around a particular value.

Coefficient

A number multiplied by a variable.

Example: The number 6 in the term $6x^2$ is a coefficient.

Collinear points

Points on the same line.

Combination

A collection of objects in no particular order.

Example: The collection 1, 2, 3 is the same combination as 3, 1, 2.

Common denominator

A number divisible by all of the denominators of two or more fractions.

Example: 12 is the common denominator of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

Common multiple

A number that is a multiple of each of two or more numbers; used to find a common denominator when operating with fractions having unlike denominators.

Example: 12 is a common multiple of 2, 3, and 4. See [multiple](#).

Commutative property of addition

The order in which two numbers are added does not affect the results. (The commutative property does not apply to subtraction.)

Examples:

$$a + b = b + a$$

$$4 + 50 = 50 + 4$$

Commutative property of multiplication

It makes no difference in which order two numbers are multiplied. (The commutative property does not apply to division.)

Examples:

$$a \cdot b = b \cdot a$$

$$3 \cdot 5 = 5 \cdot 3$$

Compare

Look for similarities and differences.

Compatible numbers

Numbers in a problem that are adjusted to make mental math easier.

Example: $16 + 11 + 24 + 35$ is adjusted so that $11 + 24 = 35$ and $35 + 35 = 70$ and $70 + 16 = 86$. So, 86 is the final answer.

Complementary angles

Two angles whose measures sum to 90 degrees.

Complementary events

Two events whose probabilities of occurring sum to one. In other words, these events are mutually exclusive and the only two things that can occur.

Example: Getting a head and getting a tail are complementary events, when flipping a coin.

Compose

To make by combining parts.

Composing numbers

Building larger units from smaller units.

Example: Ten units build one ten in base ten.

Composite number

An integer greater than one which has whole number factors other than itself and 1.

Example: 10 is a composite number because it has the factors 1, 2, 5 and 10.

Compound events

An event that consists of two or more simple events.

Example: Consider the event of rolling a cube and flipping a coin and getting a "6" and "tails".

Conclude

To make a judgment or decision after investigating or reasoning; to infer.

Conclusion

A statement that follows logically from other facts.

Conditional probability

The probability that an event will occur given that another event has already occurred.

Cone

A three-dimensional figure with one circular or elliptical base and a curved surface that joins the base to the vertex.

Examples:



Congruent figures

Figures that have the same shape and size.

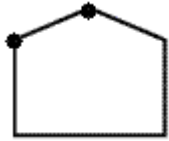
Conjecture

Inference or judgment based on inconclusive or incomplete evidence; an educated guess.

Consecutive vertices

Two vertices of a polygon that are endpoints of one side of the polygon.

Example:



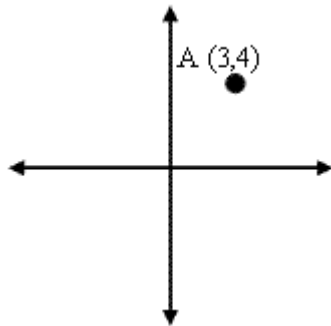
Contrast

To emphasize differences.

Coordinates

Ordered pairs of numbers that identify points on a coordinate plane.

Example: The point $(3,4)$ has an x-coordinate of 3 and a y-coordinate of 4.



Coplanar points

Points that are on the same plane.

Cube

A rectangular prism having six congruent square faces.

Cube root

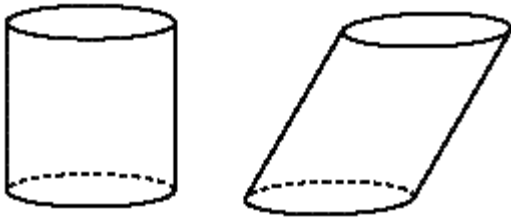
One of three (and only three) equal factors of a given number.

Example: 3 is the cube root of 27 because $3 \cdot 3 \cdot 3 = 27$.

Cylinder

A solid figure with two circular or elliptical bases that are congruent and parallel to each other.

Examples:



Data

Collected pieces of information.

Decimal number

A number expressed in base 10, such as 39.456.

Decomposing numbers

Breaking larger units into smaller units in a base.

Example: One ten breaks into ten units in base ten.

Deductive reasoning

Using logic, definitions, and other statements known to be true in order to prove another statement is true.

Denominator

The number below the fraction bar; indicates the number of equivalent pieces or sets into which something is divided.

Dependent event

An event whose probability is affected by the outcome of another event.

Derived unit of measure

A measurement determined by finding the ratio of other measurements.

Example: Density is determined by dividing the mass of quantity by its volume; speed by dividing distance covered by time elapsed.

Diagonal

A segment joining two non-consecutive vertices of a polygon.

Diagram

A drawing that represents a mathematical situation.

Diameter

A line segment (or the length of a segment) passing through the center of the circle with endpoints on the circle.

Difference

The number found when subtracting one number from another; the result of a subtraction operation; the amount by which a quantity is more or less than another number.

Digit

Any one of the ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9.

Dimension

The length, width, or height of an object.

Direct measurement

A measurement determined by use of a tool (not a calculation). See measurement.

Direct proportion

Indicates that two quantities or variables are related in a linear manner ($y = mx$). If one quantity doubles in size, so does the other; if one of the variables diminishes to 1/10 of its

former value, so does the other.

Discrete

Composed of distinct parts or discontinuous elements; a set of numbers or points that has no limit points.

Examples: Discrete — taking coins out of your pocket one at a time. Non-discrete (or continuous) — pouring water from one container to another container.

Distributive property

The product of a number and a sum is equal to the sum of the products of the number with each of the addends in the sum. That is, for all real numbers a , b , and c in a given set, $a(b + c) = ab + ac$.

Distributive property of multiplication over addition

A property of real numbers that states $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ where a , b , and c stand for any real numbers.

Example: $3(40 + 5) = (3 \cdot 40) + (3 \cdot 5)$

Dividend

A number which is to be divided by another number. Dividend \div divisor = quotient.

Examples: In $15 \div 3 = 5$, 15 is the dividend.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array} \quad \begin{array}{r} 5 \\ 3 \overline{) 15} \end{array}$$

Divisible

One integer is divisible by another non-zero integer if the quotient is an integer with remainder of zero.

Example: 12 is divisible by 3 because $12 \div 3$ is an integer, namely 4.

Division

An operation on two numbers to determine the number of sets or the size of the sets. Problems where the number of sets is unknown may be called measurement or repeated subtraction problems. Problems where the size of sets is unknown may be called fair sharing or partition problems.

Divisor

The number by which the dividend is to be divided.

Examples: In $15 \div 3 = 5$, 3 is the divisor.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array} \quad 3 \overline{) 15} \quad 5$$

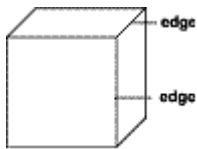
Domain

Set of all values of the independent variable of a given function, usually the x-values on a coordinate plane.

Edge

The line segment formed by the intersection of two faces of a three-dimensional figure; a cube has 12 edges.

Example:



Empirical frequency

The number of times in an experiment that a particular event occurs.

Empirical results

The results of an experiment or simulation.

Equality

Two or more sets of values are equal.

Equally likely

Two outcomes are equally likely if they have the same probability of occurring.

Equation

A mathematical sentence built from expressions using one or more equal signs.

Examples:

$$4 + 8 = 6 + 6$$

$$4 + 8 = 24 \div 2$$
$$4 + x = 12$$

Equilateral

Having congruent sides.

Equivalent fractions

Fractions that name the same number.

Example: $\frac{3}{4}$ and $\frac{6}{8}$ and $\frac{9}{12}$ are equivalent fractions.

Estimate

To find an approximate value or measurement of something without exact calculation.

Estimation

The process of finding an approximate value or measurement of something without exact calculation.

Measurement estimation — an approximate measurement found without taking an exact measurement.

Quantity estimation — an approximate number of items in a collection.

Computational estimation — a number that is an approximation of a computation that we cannot (or do not wish to) determine exactly.

Even number

Any of the integers that are exactly divisible by two.

Example: 0, 4, and 678 are even numbers.

Event

Any subset of the sample space.

Example: In rolling a number cube, the event of rolling a "3" is a singleton event because it contains only one outcome. The event of rolling an "even number" contains three outcomes.

Expanded form

A number written in component parts showing the cumulative place values of each digit in the number.

Example: $546 = 500 + 40 + 6$.

Experimental probability

The ratio of the number of times an event occurs to the number of trials.

Exponent

A numeral written above and to the right of another numeral to indicate how many times the original number is used as a factor.

Example: The exponent "3" in 4^3 means 4 is a factor 3 times; $4^3 = 4 \cdot 4 \cdot 4$.

Exponential (relationship)

Any data set or information that can be reasonably modeled by an equation of the form $y = a^x$.

Expression

A combination of variables, numbers, and symbols that represent a mathematical relationship.

Extrapolate

To estimate or approximate a value beyond a given set of data.

Face

A flat surface of a solid (3-D) figure.

Example:



Factor

One of two or more numbers that are multiplied together to obtain a product.

Example: In $4 \cdot 3 = 12$, 4 and 3 are factors.

Factorial

The product of all whole numbers from x down through 1, symbolized by $x!$.

Example: $4! = (4) (3) (2) (1) = 24$

Figure

A set of points and/or lines in 2 or 3 dimensions.

Flip

See [reflection](#).

Fluency

Understanding of mathematical procedures and the skill to use them with efficiency, accuracy, and flexibility.

Formula

A general mathematical statement, equation, or rule.

Fraction

A number that can be represented as a ratio of two real numbers.

Example:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{dividend}}{\text{divisor}} = \frac{\text{\# of parts under consideration}}{\text{\# of parts in the whole}}$$

Fraction families

Sets of fractions having denominators that are multiples of a single number.

Examples: Halves, fourths, eighths, and sixteenths are a family. Thirds, sixths, and ninths are a family.

Function (f(x))

A relation in which every value of x is associated with a unique value of y : the value of y depends on the value of x .

Example: If $y = x + 5$, then $f(x) = x + 5$.

Function machine

Applies a function rule to a set of numbers which determines a corresponding set of numbers.

Examples: 9 → Input → Rule • 7 <IMGBORDER=0SRC=' arrow.gif? Mathematics GlossaryGraphics EalrsPubDocs ealrs>Output → 63

If you apply the function rule "multiply by 7" to the values 5, 7, and 9, the corresponding

values would be:

5 → 35

7 → 49

9 → 63

Fundamental counting principle

If one event has m possible outcomes and a second independent event has n possible outcomes, then there are $m \cdot n$ total possible outcomes for the two events together.

Generalization

A conclusion reached through inductive reasoning.

Geometric sequence

A list of numbers, called terms, in which each successive term is determined by multiplying the previous term by a common factor.

Example: 1, 2, 4, 8, 16... is a geometric sequence with a first term of 1 and a common factor of 2.

Graph

A "picture" showing how certain facts are related to each other or how they compare to one another.

Greatest common factor (divisor)

The largest factor of two or more numbers; often abbreviated as GCF. The GCF is also called the greatest common divisor.

Examples: To find the GCF of 24 and 36:

1) Factors of 24 = {1, 2, 3, 4, 6, 8, 12, 24}.

2) Factors of 36 = {1, 2, 3, 4, 6, 9, 12, 18, 36}.

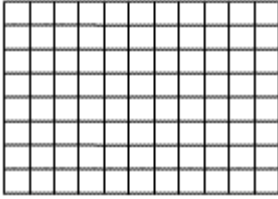
3) Common factors of 24 and 36 are {1, 2, 3, 4, 6, 12}, the largest being 12.

4) 12 is the GCF of 24 and 36.

Grid

A pattern of regularly spaced horizontal and vertical lines on a plane that can be used to locate points.

Example:



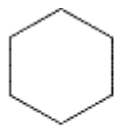
Group

To organize objects, numbers, symbols according to common characteristic(s) or to achieve a specific amount or measurement.

Hexagon

A six-sided polygon.

Examples:



regular hexagon

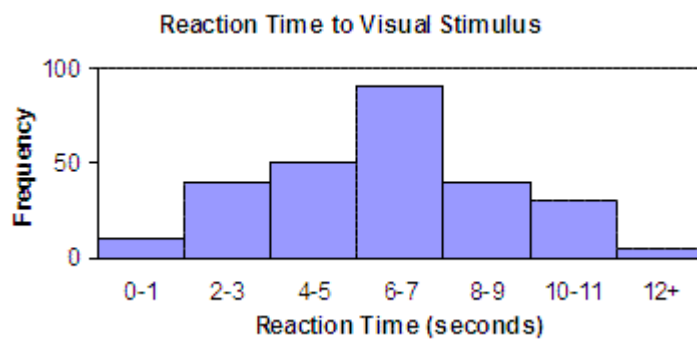


non-regular hexagons

Histogram

A graph that shows the frequency distribution for a set of data. The graph is noted for the labels of the bars being given in intervals and for no spaces between successive bars.

Example:



Horizontal

Extending side to side, parallel to the horizon.

Hypotenuse

The longest side of a right triangle (opposite the right angle).

Identity property of addition

Adding zero to a number gives a sum identical to the given number.

Identity property of multiplication

Multiplying a number by one gives a product identical to the given number.

Impossible event

An event that cannot happen, or an event with a probability of 0.

Improper fraction

A fraction in which the numerator is equal to or greater than the denominator.

Examples:

$$\frac{15}{15} \text{ and } \frac{5}{3}$$

Independent events

Two events whose outcomes have no effect on one another.

Example: The second flip of a coin is independent of the first flip of a coin.

Indirect measurement

A measurement determined without the direct application of measurement tools.

Examples:

1. Determine the likelihood a desk will pass through a doorway.
2. Which is longer, the distance around your wrist or an eraser?
3. Find a measure by the use of the Pythagorean Theorem, by similarity, or through ratios or scale factors.

Inductive reasoning

A method of reasoning in which a general statement or conjecture is made based on particular facts and/or observations.

Example: Deriving a general rule to describe a set of numbers from an observed pattern.

Inequality

Two or more sets of values that are not equal.

Infer

To draw a conclusion from facts or evidence.

Inference

A conjecture based on inductive reasoning.

Integer

The counting numbers (1, 2, 3,...), their opposites (-1, -2, -3,...), and zero.

Integral

Refers to being an integer.

Interpolate

To estimate or approximate a value between two given values.

Interpret

To explain the meaning of information, facts, and/or observation.

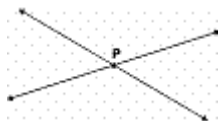
Intersect

To meet or cross.

Intersecting lines

Lines that meet at a point.

Example:



Interval

Spacing of (or space between) two numbers on a number line.

Inverse property of multiplication

Each non-zero real number x has a multiplicative inverse, denoted by $\frac{1}{x}$, such that their product is 1.

Example: The number 3 has a multiplicative inverse of $\frac{1}{3}$.

Inverse proportion

Two quantities are inversely proportional when increase in one corresponds to decrease in the other. If one quantity doubles in size the other is halved. If one quantity is multiplied by ten the other is divided by ten.

Example: If the time spent traveling a certain distance is doubled, the speed of travel halved.

Investigate

To research using careful observation and examination of the facts; to inquire.

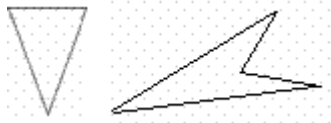
Irrational number

A number that cannot be written as a ratio of two integers. The decimal extensions of irrational numbers never terminate and never repeat.

Irregular polygon

A polygon whose interior angles are not equal and/or its sides are not equal in length.

Examples:



Isosceles triangle

A triangle with two congruent sides; an alternate definition is a triangle with at least two congruent sides (there is no common agreement on a definition of an isosceles triangle).

Justify

To prove or show to be true or valid using logic and/or evidence.

Least common multiple

The smallest positive multiple of two or more integers.

Example: 12 is the LCM of 2, 3 and 4, because it is the smallest number that is a multiple of all three numbers. 12 is also the LCM of 2, -3, 4.

Less likely

An event or outcome that has a probability of occurring less than the probability of another event or outcome.

Likely

Probably will happen.

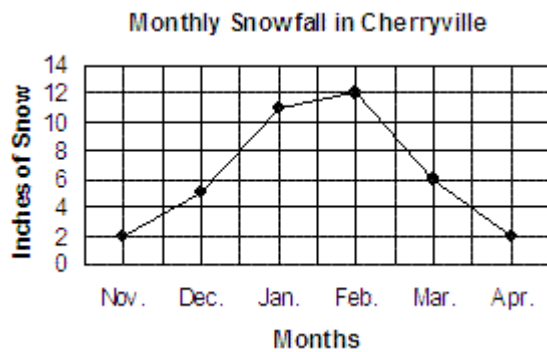
Line

See undefined term.

Line graph

A graph that uses lines (segments) to show that something is increasing, decreasing, or staying the same over time.

Example:



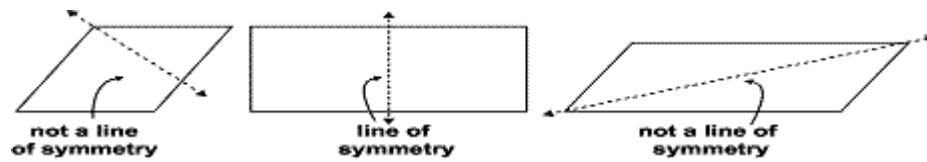
Line of best fit

A line drawn on a scatter plot to estimate the linear relation between two sets of data.

Line of symmetry

A line on which a figure can be folded into two parts that are congruent mirror images of each other.

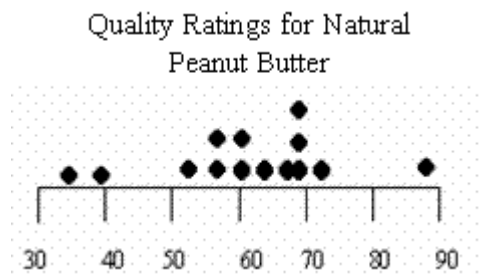
Examples:



Line plot

A line plot, sometimes called a dot plot, starts with a line that represents one variable. The values of the variable are labels on the line. Each observation is marked as a point above the line.

Example:



Line segment

A part of a line defined by two endpoints and all the points of the line between them.

Linear equation

An equation whose graph on a coordinate grid is a line. The equation can be written in the form

$$y = mx + b.$$

Linear inequality

An inequality whose graph on a coordinate grid is bounded by a line. The inequality can be written in the form $y (\leq, <, >, \text{ or } \geq) mx + b$.

Linear model

An equation that may be expressed as $y = mx + b$ to exactly or nearly represent a data set.

Linear or linear relation

Any data set or information that could be reasonably modeled with a line.

Location

The position on a number line, coordinate plane, or 3-dimensional space where an object is or can be found.

Lower quartile

The median of the lower half of an ordered set of data.

Mass

The amount of matter in an object.

Mean

A measure of central tendency found by summing the members of a set of data and dividing the sum by the number of members of the set (also called the arithmetic mean).
Example: If $A = 20$ children, $B = 29$ children, and $C = 26$ children, the mean number of children is found by summing the three numbers $20 + 29 + 26 = 75$ and then dividing the sum, 75, by the number 3. So, 25 is the mean of 20, 29, 26.

Measurement

The numerical amount associated with dimensions, quantity, length, or capacity.
Example: The length of a line segment and the volume of a cube are measurements.

Measures of central tendency

Numbers that give some indication of the distribution of data. Three common measures of central tendency are mean, median, and mode.

Median

The number in the middle of a set of data arranged in order from least to greatest or from greatest to least; or the average of the two middle terms if there is an even number of terms.

Examples:

- 1) For the data: 6, 14, 23, 46, 69, 72, 94 → the median is 46 (the middle number).
- 2) For the data: 6, 14, 23, 61, 72, 94 → the median is 42 (the average of the two middle

numbers in the list).

Method

A systematic way of accomplishing a task.

Minuend

In subtraction, the minuend is the number you subtract from.

Example:

$$\begin{array}{r} 90,000 \text{ (minuend)} \\ - 3,456 \text{ (subtrahend)} \\ \hline 86,544 \text{ (difference)} \end{array}$$

Mixed number

A number expressed as the sum of an integer and a proper fraction; having a whole part and a fractional part.

Example: $6\frac{2}{3}$

Mode

The item that occurs most frequently in a set of data. There may be one, more than one, or no mode.

Example: The mode in {1, 3, 4, 5, 7, 7, 9} is 7.

More likely

An event or outcome that has a probability of occurring greater than the probability of another event or outcome.

Multiple

A multiple of a number is the product of that number and an integer.

Examples: Multiples of 2 = {2, 4, 6, 8, 10, 12, ...}.

Multiples of 3 = {3, 6, 9, 12, ...}.

Multiples of 4 = {4, 8, 12, ...}.

Multiple transformations

A combination of transformations applied sequentially to a figure.

Example: Reflection of one figure over one line followed by reflection over a second line or, translation of one figure on a graph followed by a reflection over an axis.

Multiplicand

In multiplication, the multiplicand is the factor being multiplied.

Example:

$$\begin{array}{r} 83.9 \text{ (multiplicand)} \\ \times \underline{3.06} \text{ (multiplier)} \\ \hline 256.734 \text{ (product)} \end{array}$$

Multiplication

An operation on two numbers that tells how many in all. The first number is the number of sets and the second number tells how many in each set. Problem formats can be expressed as repeated addition, fair shares, an array approach or a Cartesian product approach.

Multiplier

In multiplication, the multiplier is the factor being multiplied by.

Example:

$$\begin{array}{r} 83.9 \text{ (multiplicand)} \\ \times \underline{3.06} \text{ (multiplier)} \\ \hline 256.734 \text{ (product)} \end{array}$$

Mutually exclusive

Two events are mutually exclusive if it is not possible for both of them to occur at the same time.

Example: If a die is rolled, the event "getting a 1" and the event "getting a 2" are mutually exclusive since it is not possible for the die to be both a one and a two on the same roll.

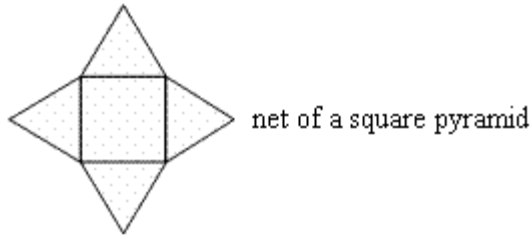
Natural number

A number from the set of numbers $\{1, 2, 3, 4, \dots\}$. The natural numbers are also called the counting numbers or positive integers.

Net

A representation of a three-dimensional figure that is unfolded.

Example:



Non-coplanar

A set of points in space that cannot be contained in the same plane.

Non-linear

A data set or function that, when plotted, does not have the characteristics of a line.

Non-repeating decimal

A decimal number in which there are no digits that endlessly repeat a pattern.

Non-standard units of measure

Measurement units that are not commonly accepted as standard but are applied uniformly when measuring.

Example: Paperclips, pencils, cubes

Number

A symbol that represents an amount or measurement.

Number line

A line that shows numbers ordered by magnitude; an arrowhead at each end indicates that the line continues endlessly in both directions; equal intervals are marked and labeled.

Number pattern

A repeated sequence of numbers or a sequence of numbers created by following a particular rule.

Number periods

A group of three places used for the digits in large numbers (e.g. ones, thousands, millions,

billions).

Numeral

A symbol (not a variable) used to represent a number.

Numerator

The number above the line in a fraction; indicates the number of parts being considered.

Obtuse angle

An angle with measure greater than 90 degrees and less than 180 degrees.

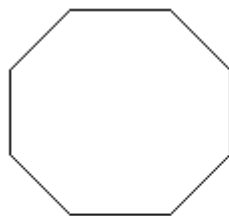
Obtuse triangle

A triangle with one obtuse angle.

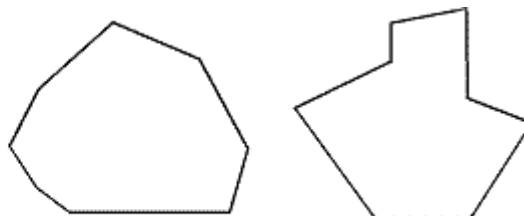
Octagon

An eight-sided polygon.

Examples:



regular octagon



non-regular octagons

Odd number

Integers that are not divisible by two.

Examples: 53 and 701 are odd numbers.

Odds

A ratio of probabilities in favor of, or against the occurrence of an event. The odds in favor = $P(\text{an event can occur}) : P(\text{an event cannot occur})$. The odds against = $P(\text{an event cannot occur}) : P(\text{an event can occur})$.

One-dimensional

A shape (geometric figure) having only length; no width or height (a line or line segment).

Open-ended problem

A problem with different possible solution paths and which may have different solutions depending on the route taken.

Operation

A mathematical process that combines numbers; basic operations of arithmetic include addition, subtraction, multiplication, and division.

Order

To organize according to size, amount, or value.

Order of operations

In simplifying an expression involving a number of indicated operations, perform the operations in the following order:

1. Complete all operations that are grouped with grouping symbols.
2. Calculate powers and roots and in the order they occur from left to right.
3. Calculate all multiplications and divisions left to right.
4. Calculate all additions and subtractions left to right.

Examples:

$$7 + 3 \cdot 8 = 31 \text{ [multiply } 3 \cdot 8 \text{ before adding } 7]$$

$$(7 + 3) \cdot 8 = 80 \text{ [add } 7 \text{ and } 3 \text{ before multiplying by } 8]$$

$$7 + 3^2 \cdot 8 = 79 \text{ [square } 3, \text{ multiply by } 8, \text{ and then add } 7]$$

Ordered pair

Two numbers (elements), for which order is important. When used to locate points on a coordinate graph the first element indicates distance along the x-axis (horizontal) and the second indicates distance along the y-axis (vertical). See illustration for coordinates.

Ordinal number

A number that designates the position of an object in order; first, second, and third are examples of ordinal numbers.

Examples: Eraser is the second element in the set {pencil, eraser, desk, chalkboard, book, file, paper}; z is the twenty-sixth element in the set {a, b, c, d, ...z}.

Origin

Zero on a number-line or the point (0,0) on a coordinate plane.

Outcome

One of the possible results in a probability situation or activity.

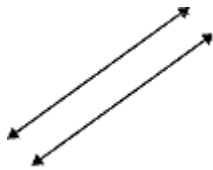
Outlier

A number in a set of data that is much larger or smaller than most of the other numbers in the set.

Parallel lines

Lines that lie in the same plane and never intersect.

Example:



Parallelogram

A quadrilateral with opposite sides parallel.

Example:



Pattern

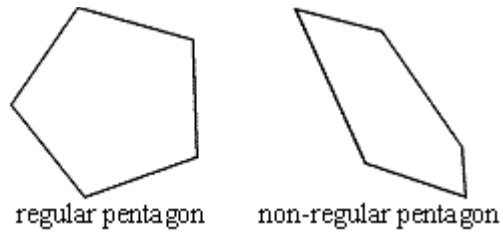
The arrangement of numbers, pictures, etc., in an organized and predictable way.

Example: 3, 6, 9, 12,.... or \square , \triangle , \square , \triangle , \square , \triangle .

Pentagon

A five-sided polygon.

Examples:



Percent

A ratio of a number to 100. Percent means per hundred and is represented by the symbol %.

Example: "35 to 100" means 35%.

Perimeter

The distance around the outside of a shape or figure.

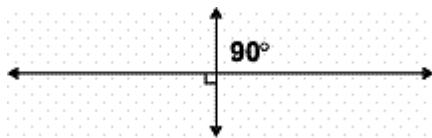
Perimeter of polygon

The sum of the lengths of the sides of the polygon.

Perpendicular lines

Lines that lie on the same plane that intersect to form right angles.

Example:



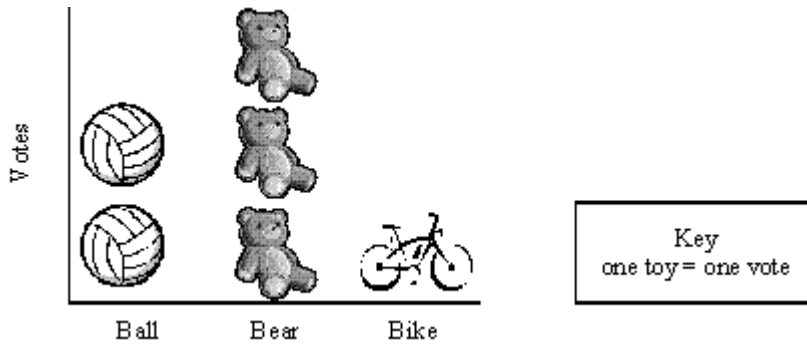
Pi

The ratio of the circumference to the diameter of the same circle. The value of pi is approximately 3.14159 and is represented by the symbol π .

Pictograph

Graph that uses pictures or symbols to represent similar data.

Example:



Place value

The value of a digit as determined by its place in a number.

Example: In the number 135, the 3 means $3 \cdot 10$ or 30. In the number 356, the 3 means 3 hundreds or 300.

Plane

See undefined term.

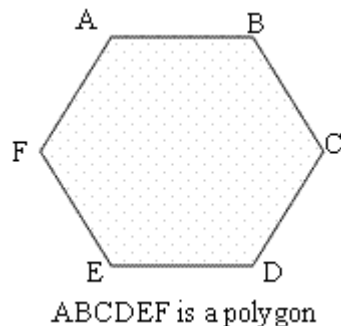
Point

See undefined term. Although this is an undefined term it is helpful to visualize a point as the intersection of two lines.

Polygon

A closed plane figure having three or more straight sides.

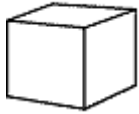
Example:



Polyhedron

A solid (3-D) figure, the faces of which are polygons.

Example:



Population

A group of people, objects, or events that fit a particular description.

Possible outcomes

All of the outcomes that can occur.

Example: If there are three colors on a spinner, each color is a possible outcome.

Power

A number (exponent) representing repeated multiplication.

Example: In 3^4 , 4 is a power of 3 that represents repeated multiplication so that $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$.

Precision

An indication of how finely a measurement is made; related to the unit of measurement and the calibration of the tool.

Predict

To tell about or make known in advance, especially on the basis of special knowledge or inference.

Prediction

A prediction is a description of what will happen before it happens. It is a foretelling that is based on a scientific law or mathematical model.

Prime number

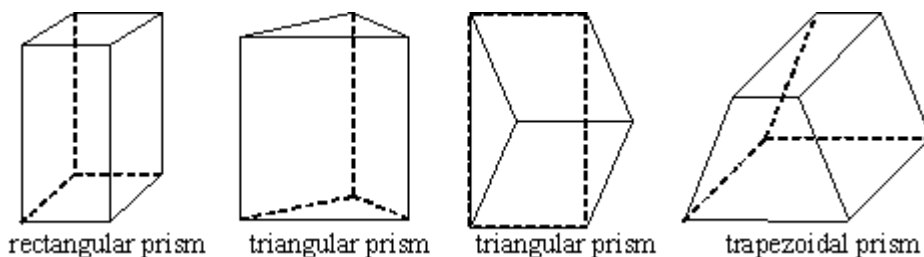
A whole number greater than 1 having exactly two whole number factors, itself and 1.

Example: 7 is prime since its only whole number factors are 1 and 7. 1 is not a prime number.

Prism

A 3-dimensional figure that has 2 congruent and parallel faces (bases) that are polygons and the remaining (lateral) faces are parallelograms.

Examples:



Probability

The numerical measure of the chance that a particular event will occur, depending on the possible events. The probability of an event, $P(E)$, is always between 0 and 1, with 0 meaning that there is no chance of occurrence and 1 meaning a certainty of occurrence.

Product

The result of a multiplication expression; factor \cdot factor = product.

Example: $3 \cdot 4 = 12$, 12 is the product.

Proof

A logical argument that a specified statement is true based on assumed statements and previously determined true statements.

Proper fraction

A fraction, the absolute value of whose numerator is an integer less than its integer denominator.

Examples: $\frac{15}{22}$ is a proper fraction; $\frac{22}{15}$ is not. $\frac{2}{5}$ is a proper fraction; $\frac{5}{2}$ is not.

Proportion

An equation showing that two ratios are equivalent.

Example: $\frac{2}{3} = \frac{6}{9}$

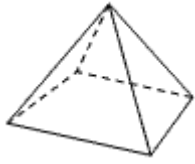
Proportional

Constituting a proportion; having the same, or a constant, ratio; as proportional quantities.

Pyramid

A solid (3-D) figure whose base is a polygon and whose other faces are triangles that meet at a common point (vertex).

Example:



Pythagorean theorem

In any right triangle having a hypotenuse of length c and two legs of lengths a and b ,
 $a^2 + b^2 = c^2$.

Quadrants

The four sections of a coordinate grid that are separated by the axes. By convention, they are numbered counterclockwise from upper right, I, II, III, IV.

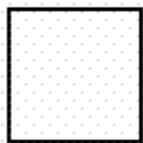
Quadratic equation

An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$.

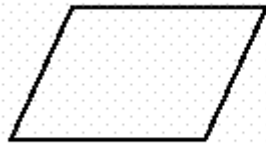
Quadrilateral

A four-sided polygon.

Example:



square



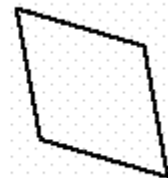
parallelogram



trapezoid



rectangle



rhombus

Questionnaire

A set of questions for a survey.

Quotient

The result of dividing one number by another number. Dividend \div divisor = quotient.

Examples: In $15 \div 3 = 5$, 5 is the quotient.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array} \qquad \begin{array}{r} 5 \\ 3 \overline{) 15} \end{array}$$

Radius

The segment, or length of the segment, from the center of a circle to a point on the circle.

Random sample

A sample in which every person, object, or event in the population has the same chance of being selected for the sample.

Range (functional)

The set of all values of the dependent variable of a given function, usually the y-value on a coordinate plane.

Range (statistical)

The absolute value of the difference between the largest and smallest values in a set of data.

Example: The range of $\{2, 4, 6, 7, 9, 13\}$ is $|2-13|$ or $13-2$ or 11.

Rate

A ratio comparing two different quantities.

Examples: Miles per hour, gallons per mile, and heartbeats per minute are rates.

Ratio

A comparison of two numbers using a variety of written forms.

Examples: The ratio of two and five may be written "2 to 5" or 2:5 or $\frac{2}{5}$.

Rational number

Any number that can be expressed as a ratio of two integers, with the denominator non-zero.

Examples: 34 can be written $\frac{34}{1}$, 4.32 can be written as $\frac{432}{100}$, $3\frac{1}{2}$ can be written as $\frac{7}{2}$.

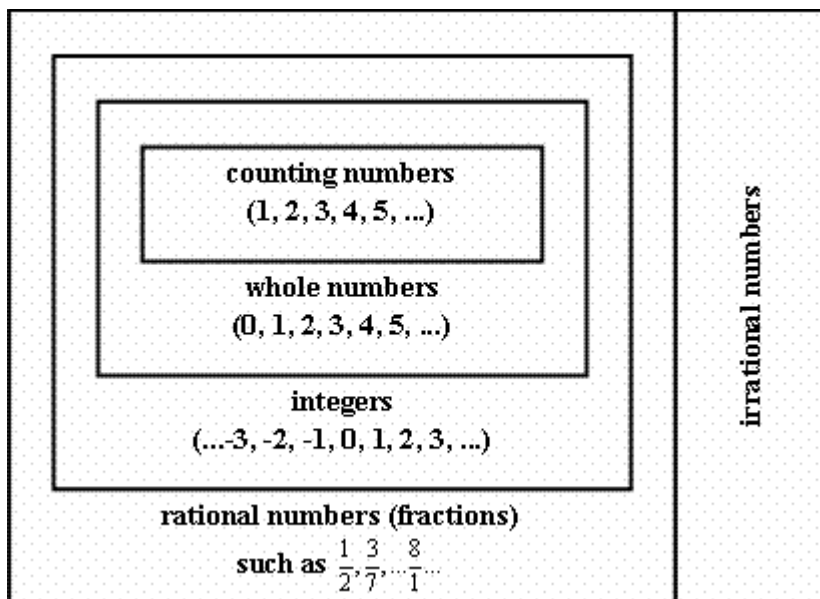
Ray

A part of a line that has one end point and extends indefinitely in one direction.

Real number

The combined set of the rational and irrational numbers.

Real number system



Reasonable

Within likely bounds; sensible.

Reciprocal

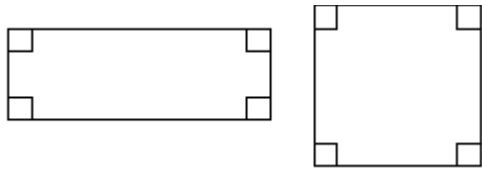
Two numbers that have a product of 1.

Example: $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals because $\frac{3}{4} \cdot \frac{4}{3} = 1$.

Rectangle

A quadrilateral with four right angles. A square is a rectangle.

Examples:



Rectangular prism

A prism with 2 rectangular bases.

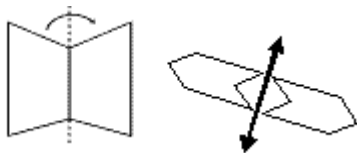
Examples:



Reflection or reflection on a line

A transformation of a figure by flipping the figure over a line, creating a mirror image.

Examples:



Regroup

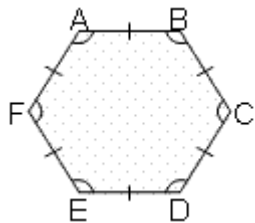
Use place value to think of a number in a different way to make addition and subtraction easier.

Example: 425 can be thought of as $300 + 120 + 5$ in order to make subtracting 172 easier.

Regular polygon

A polygon with all sides having the same length and all angles having the same measure.

Example: ABCDEF is a regular polygon called a hexagon.



Relative value

A comparison based on size, amount or value.

Remainder

The undivided part that is left after division; it is less than the divisor.

Example:

$$\begin{array}{r} 5 \text{ r } 2 \\ 6 \overline{) 32} \\ \underline{30} \\ 2 \end{array}$$

Repeating decimal

A decimal number whose expression contains a repeating pattern of decimals from some point in the expression forward.

Example: 3.121212... is a repeating decimal with the pattern of the digits "12". This decimal can be written as $3.\overline{12}$.

Represent

To present clearly; describe; show.

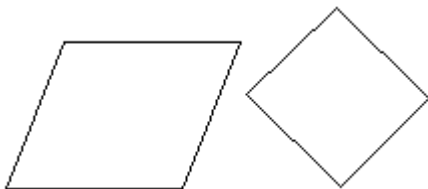
Revise

To change or modify based on reflection and evaluation.

Rhombus

A quadrilateral with all four sides equal in length.

Examples:



Right angle

An angle whose measure is 90 degrees. See [angle](#) and [triangle](#).

Right circular cylinder

A cylinder whose bases are circles and the centers of whose sections form a line cylinder perpendicular to the bases.

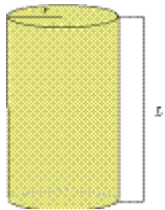
Example:



Right cylinder

A cylinder with centers of whose sections form a line perpendicular to the bases.

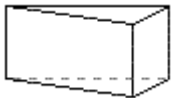
Example:



Right prism

A prism with rectangular lateral faces (sides).

Example:



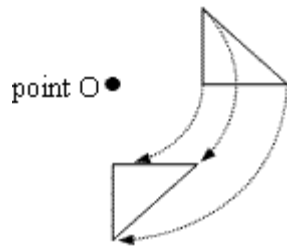
Right triangle

A triangle having one right angle. See [angle](#) and [triangle](#).

Rotation

A transformation of a figure (or points) in a plane resulting from turning a figure around a center point O either clockwise counterclockwise.

Example:



Rule

A procedure; a prescribed method; a way of describing the relationship between two sets of numbers.

Example: In the following data, the rule is to add 3.

<u>Input</u>	<u>Output</u>
3	6
5	8
9	12

Sample

A portion of a population or set used in statistics.

Example: All boys under the age of ten constitute a sample of the population of all male children.

Sample space

A set of all possible outcomes to a specified experiment.

Scale

Sequenced collinear marks, usually at regular intervals or else representing equal steps, that are used as a reference in making measurements.

Scale factor

A ratio that compares two sets of measurements such as the size of a model to the actual size of the object being modeled.

Scalene polygon

A polygon with no two sides equal.

Scatter plot

A graph of points (x, y) , one for each item being measured, on a coordinate plane. The two coordinates of a point represent their observed, paired values.

Example: The ordered pairs may relate temperature to time of day (time, temp).

Scientific notation

A number expressed in the form of $a \cdot 10^n$ where $1 \leq a < 10$ and n is an integer.

Example: 342.15 is written in scientific notation as $3.4215 \cdot 10^2$.

Sequence

A set of numbers arranged in a special order or pattern.

Series

The indicated sum or difference of a sequence of numbers.

Example: The series $1 + 3 + 5 + \dots + 19$ is the series of the first ten odd whole numbers.

Set

A group of objects.

Side

A line segment connected to other segments to form the boundary of a polygon.

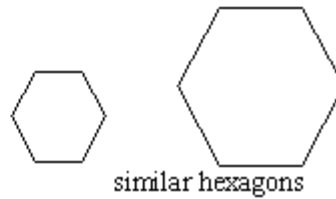
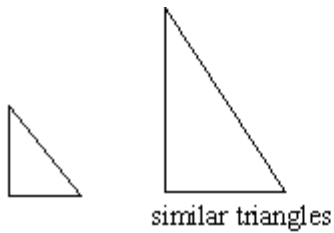
Example:



Similar figures

Having the same shape but not necessarily the same size (congruent corresponding angles and proportional corresponding sides).

Examples:



Similarity

Characteristic of similar figures.

Simulation (probability)

An experiment to model a real-world situation.

Example: Toss a coin to model true-false; heads = true, tails = false.

Single variable equation

An equation with one variable (must have an "=" sign).

Example: $3x + 2 = 8$

Single variable expression

An expression with one variable (must not have an "=" or inequality sign).

Example: $3x + 2$

Single variable inequalities

An inequality with one variable (must use $<$, \leq , $>$, \geq or \neq).

Example: $3x + 2 > 8$

Skip count

Counting by groups as in skip count by 2s, 3s, 4s, ..., 12s. Can be thought of as a precursor to multiplication.

Slide

See [translation](#).

Slope

The ratio of the change in y-units (vertical) to the change in x-units (horizontal) between

two points on a line.

Example: The slope of a line through (3,4) and (9,5) is $\frac{5-4}{9-3}$ or $\frac{1}{6}$.

Solid

A geometric figure with three dimensions.

Solution of an equation

A number that, when substituted for the variable in an equation, results in a true statement.

Solve

To find the solution to an equation or problem.

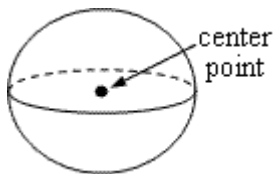
Spatial

Of, pertaining to, involving, or having the nature of space or three-dimensions.

Sphere

A closed surface consisting of all points in space that are the same distance from a given point (the center).

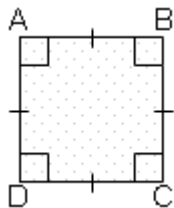
Example: A basketball.



Square

A rectangle with congruent sides. See rectangle.

Example:



Square number

An integer that is a square of another integer.

Example: 49 is a square number because 49 is the square of 7. ($49 = 7 \cdot 7$)

Square root

One of two equal non-negative factors of a given real number.

Example: 7 is the square root of 49 because $7 \cdot 7 = 49$.

Square unit

A unit, such as a square meter (m^2) or a square foot (ft^2), used to measure area.

Standard form of a number

A number written with one digit for each place value.

Examples: The standard form for five hundred forty-six is 546. The standard form for three thousand six is 3,006.

Standard units of measure

Units of measure commonly used, generally classified in the U.S. customary system or metric system.

Example: feet, meters, acres, gallons, liters

Stem-and-leaf plot

A method of organizing a list of numbers into stems and leaves where leaves represent units and stems represent the other digits. Stems are listed in increasing or decreasing order. Leaves are associated with their stem but need not be sequential.

Examples:

Ages of Adults in the Park

Data Set	Stem	Leaves
23 25 29	2	3 5 9 9
36 38 39	3	6 8 9 9
52 54 55	5	2 4 5 5

Key: 2 | 3 represents 23 years

Number of Customers for Dinner

Data Set	Stem	Leaves
120 126 124	12	6
135 133	13	5
142 140 143	14	3

Key: 2 | 3 represents 23 years

Strategy

A systematic plan used in problem solving; such as looking for a pattern, drawing a

diagram, working backward, etc.

Subtraction

An operation that is a removal of sets from an initial set; or finds the difference between two amounts when comparing two quantities.

Subtrahend

In subtraction, the subtrahend is the number being subtracted.

Example:

$$\begin{array}{r} 90,000 \text{ (minuend)} \\ - 3,456 \text{ (subtrahend)} \\ \hline 86,544 \text{ (difference)} \end{array}$$

Successive events

Events that follow one another in a compound probability setting.

Sum

The result of addition. Addend + Addend = Sum

Example: The sum of 32 and 46 is 78.

Summary

A series of statements containing evidence, facts, and/or procedures that support a result.

Support

To uphold or argue as valid or correct.

Surface area

The sum of the areas of all of the faces (or surfaces) of a 3-D object. Also the area of a net of a 3-D object. Calculations of surface area are in square units (in², m², or cm²).

Survey

To get an overview by gathering data.

Symbol

A letter or sign used to represent a number, function, variable, operation, quantity, or relationship.

Examples: a , $=$, $+$, \dots

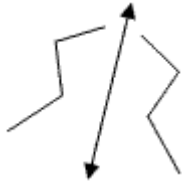
Symmetrical

Having a line, plane, or point of symmetry such that for each point on the figure, there is a corresponding point that is the reflection of that point. See [line of symmetry](#).

Symmetry (line)

The geometric property of being balanced about a line.

Example: A figure is symmetric with respect to a line, called the axis of symmetry, if it can be folded on the line and the two halves of the figure are congruent and match.



Symmetry (rotation)

A figure is symmetric about a point if there is a rotation of the figure of less than 360 degrees about the point that allows the figure to correspond with itself.

System of equations

Two or more equations in terms of the same variables. The solution of a system is a set of values for the unknowns (variables) that satisfy all the equations simultaneously.

Example: Given the system of two equations $x + y = 3$ and $2x - y = 0$; $(x,y) = (1,2)$ is the solution for the system because $(1,2)$ is a solution for both $x + y = 3$ and $2x - y = 0$.

T-chart

A table of two sets of values; an input-output table.

Example:

Input	Output
1	4
2	5
3	6

Table

A method of displaying data in rows and columns.

Terminating decimal

A terminating decimal is a fraction whose decimal representation contains a finite number of digits.

Example: $\frac{1}{4} = .25$

Tessellate

To form or arrange congruent figures (sometimes polygons) in a pattern on a plane with no gaps and no overlaps.

Theoretical probability

Measure of the likelihood that an event will occur; the ratio of favorable outcomes to the number of possible outcomes.

Example: Knowing that there are six possible outcomes for rolling a fair number cube one can assign the probability of $\frac{1}{6}$ to each of the possible outcomes.

Three-dimensional figure (3-D figure)

A closed shape in space having length, width, and height.

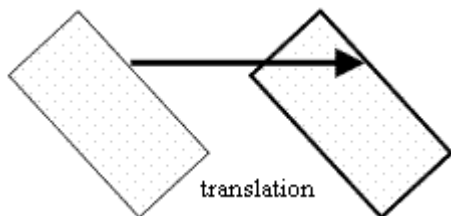
Transformation (geometric)

A change in position/location of a figure. Types of transformations include translation (slide), reflection (flip), rotation (turn), (or combinations of these).

Translation

A transformation of a figure by sliding without turning or flipping in any direction.

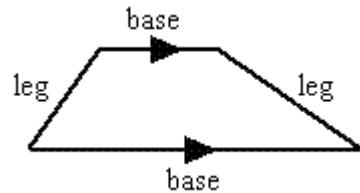
Example:



Trapezoid

A quadrilateral that has exactly two parallel sides; an alternate definition is a quadrilateral with at least two parallel sides. (There is no common agreement on a definition of a trapezoid.)

Example:



Trend

The general direction or tendency of a set of data.

Triangle

A three-sided polygon.

Turn

See [rotation](#).

Two-dimensional figure (2-D figure)

A shape (geometric figure) having length and width. (A flat figure.)

Undefined term

A term whose meaning is not defined in terms of other mathematical words, but instead is accepted with an intuitive understanding of what the term represents. The words "point," "line," and "plane" are undefined terms from geometry.

Unique

Means there is one and only one object or result.

Example: The product of two integers is unique.

Univariate data

Data involving one variable.

Example: A list of test scores.

Unknown

In algebra, the quantity represented by a variable.

Valid statement

A statement taken as being true in a reasoning situation.

Validate

To determine, substantiate, verify, or confirm whether a given statement or argument passes specific standards.

Variability of data

Range, average deviation, standard deviation, and spread are all ways to describe the variability of data.

Variable

A symbol used to represent a quantity that changes or can have different values.

Example: In $5n$, the n is a variable.

Variation (direct)

A relationship between two variables that can be expressed in the form $y = kx$ where $k \neq 0$; $y = kx$ can be read as "y varies directly with respect to x".

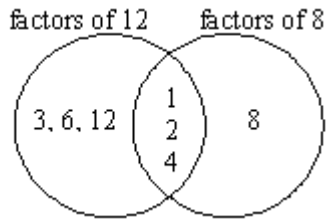
Variation (inverse)

A relationship between two variables that can be expressed in the $y = \frac{k}{x}$ where $k \neq 0$. $y = \frac{k}{x}$ can be read as "y varies inversely with respect to x".

Venn diagram

A diagram that shows relationships among sets of objects.

Example:



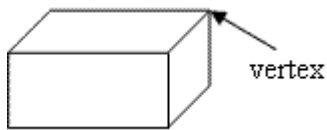
Verify

To establish as true by presentation of evidence.

Vertex

A point at which two line segments, lines, or rays meet to form an angle, where edges of a polygon or polyhedron intersect, or the point opposite the base in a pyramid or cone.

Example:



Vertical

Extending straight up and down; perpendicular to the horizon.

Examples:

- 1) A power pole is vertical to the ground.
- 2) Lines drawn on paper from top to bottom that are parallel to the sides of the paper represent vertical lines.

Vertices

Plural of vertex.

Volume

A measure in cubic units of the space contained in the interior of a solid figure.

Example: The number of cubic units contained in the interior of a rectangular solid.

Weight

A measure of the heaviness of, or the force of, gravity on an object.

Whole number

A number from the set of numbers $\{0, 1, 2, 3, 4, \dots\}$.

Word form

The expression of numbers and/or symbols in words.

Example: 546 is "five hundred forty-six". The "<" symbol means "is less than". The ">" symbol means "is greater than". The "=" symbol means "equals" or "is equal to".

X-axis

One of two intersecting straight (number) lines that determine a coordinate system in a plane; typically the horizontal axis.

Y-axis

One of two intersecting straight (number) lines that determine a coordinate system in a plane; typically the vertical axis.

Zero property of addition

Adding zero to a number gives a sum identical to the original number. Zero is the identity element of addition. See identity property.

Examples: $4 + 0 = 4$ and $56.89 + 0 = 56.89$

Zero property of multiplication

The product of any number and zero is zero.

Examples: $4 \cdot 0 = 0$ and $0 \cdot 456.7 = 0$